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# Dynamic analysis of bridge with non-Gaussian uncertainties under a moving vehicle

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ABSTRACT

An analysis method on the bridge–vehicle interaction problem with uncertainties is proposed. The bridge is modeled as a simply supported Euler–Bernoulli beam with non-Gaussian material parameters with a vehicle moving on top modeled by a deterministic four degrees-of-freedom mass–spring system. The non-Gaussian uncertainty in bridge is modeled by the Spectral Stochastic Finite Element Method (Ghanem and Spanos (1991) [17]), and the mathematical model of the coupled bridge–vehicle system, with the road surface roughness assumed as a Gaussian random process, will be solved by the *Newmark-$\beta$* method. The proposed model is verified by the Monte Carlo Simulation with numerical examples. Different levels of uncertainties in both the excitation and system parameters are investigated. Criteria on the selection of both the order of Polynomial Chaos and the threshold for truncation in the Karhunen–Loève expansion are provided. Results show that the proposed algorithm is promising for the dynamic analysis of the bridge–vehicle interaction problem even with a high level of system and excitation uncertainties.



## 1. Introduction

The dynamic responses of a bridge structure subject to moving vehicular loads have been studied for decades. The research on the bridge–vehicle interaction problem can be mainly categorized into two kinds according to the technique employed to solve the equation of motion of the bridge–vehicle system: (1) methods based on modal superposition technique [1–5]; and (2) methods based on finite element method [6–8]. The latter is capable of handling more complex bridge–vehicle models with complex boundary conditions compared with the former which needs vibration mode shapes for solving system equations.

The above methods are based on deterministic system parameters of the bridge and deterministic excitation due to moving vehicle. Moreover, the road surface roughness is considered as deterministic samples of irregular profile according to its power spectral density defined in the ISO standard [9]. However, the contact forces between the bridge and the vehicle are random due to the road surface roughness and also the bridge–vehicle system often exhibits an inherent randomness in the system parameters such as Young's modulus, mass density, etc. especially when there are local changes in the structure. The conventional deterministic analysis generally represents only an "approximation" of the actual reality due to these inherent uncertainties in the structural properties as well as in the loading processes. A stochastic analysis should be adopted to study the bridge–vehicle interaction problem.

Stochastic excitations are often modeled as random processes and different tools have been developed for representing these processes [10] such as spectral representation [11], Karhunen–Loève (K–L) expansion [12], Polynomial Chaos (PC) expansion [13], etc. Currently the stochastic analysis of a structural system with uncertain system parameters is usually performed with the Monte Carlo Simulation [14] which is very versatile but comparatively time-consuming. This technique often serves to validate other approximate analytical methods. One of the other alternate approaches widely used for evaluating the stochastic response is the perturbation approach [15]. However, this approach is accurate only when the random parameters have small deviations from the center value and requires simulations to assess the reliability of the results. The Neumann expansion method [16], which is similar to the perturbation method, also requires simulations to assess the reliability of the results. The convergence of the Neumann series to represent the inverse operator requires the norm of the kernel smaller than unity. The Spectral Stochastic Finite Element Method (SSFEM) [17,18] has been developed to overcome these weaknesses. It is a general technique for the solution of complex problems in probabilistic mechanics and is capable of handling system uncertainties with a large range of variation when the K–L expansion and the PC expansion are employed. Numerous applications [19,20] can be found with the SSFEM employed for solving the engineering problem with uncertainties.

Based on the theory developed for stochastic analysis in recent years, research on the dynamic response of a bridge deck

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under moving vehicle (or forces) has been performed by many researchers. Some researchers only considered the randomness in the excitation due to the road surface roughness where the system parameters of both bridge and vehicle were treated as deterministic. These works can mainly be classified into the frequency domain method [21,22] and the time domain method [23,24]. Others extended the work by introducing randomness in the system modeling [25–28] in which the Gaussian assumption was made on the system parameters and perturbation method was employed for the solution. However, when the variation of uncertainty increases, the Gaussian assumption on the system parameters, which has a very small probability to take up a negative value, may lead to an inaccurate solution and the perturbation method also tends to become less accurate.

In this paper, an extension of the authors' previous work [29] is proposed to analyze the bridge–vehicle interaction problem with uncertainties. The bridge is modeled as a simply supported Euler–Bernoulli beam with non-Gaussian material parameters with a vehicle moving on top modeled by a deterministic four degrees-of-freedom mass–spring system. The non-Gaussian uncertainty in the bridge is modeled by the Spectral Stochastic Finite Element Method (SSFEM), and the mathematical model of the coupled bridge–vehicle system, with the road surface roughness assumed as a Gaussian random process, will be solved by the *Newmark-β* method. The proposed model is verified by the Monte Carlo Simulation with numerical examples. Different levels of uncertainties in both the excitation and system parameters are investigated. Criteria on the selection of both the order of Polynomial Chaos and the threshold for truncation in the Karhunen–Loève expansion are provided. Results show that the proposed algorithm is promising for the dynamic analysis of the bridge–vehicle interaction problem even with a high level of system and excitation uncertainties.

The outline of the present work is as follows: the equation of motion of the bridge–vehicle system with uncertainties is derived in Section 2. Since the Karhunen–Loève expansion and Polynomial Chaos expansion are required for representing the random processes, the basic theories of these expansions are introduced in Sections 3 and 4, respectively. The modeling of the bridge–vehicle interaction problem with non-Gaussian system uncertainties using SSFEM is given in Section 5 together with discussions on the response statistics. Numerical simulations are presented in Section 6 to verify the effectiveness of the proposed algorithm with discussions on different levels of system and excitation randomness. Some conclusions are drawn in Section 7.

## 2. The system equation of motion

A vehicle with four degrees-of-freedom moving at a uniform speed $v$ over a simply supported bridge deck is shown in Fig. 1. The equation of motion of the vehicle is derived using the Lagrange formulation as follows:

$$\begin{bmatrix}\mathbf{M}_{V1} & 0\\ 0 & \mathbf{M}_{V2}\end{bmatrix}\ddot{\mathbf{Y}}+\begin{bmatrix}\mathbf{C}_{V11} & \mathbf{C}_{V12}\\ \mathbf{C}_{V21} & \mathbf{C}_{V22}\end{bmatrix}\dot{\mathbf{Y}}+\begin{bmatrix}\mathbf{K}_{V11} & \mathbf{K}_{V12}\\ \mathbf{K}_{V21} & \mathbf{K}_{V22}\end{bmatrix}\mathbf{Y} = -\begin{Bmatrix}0\\ \mathbf{F}(t,\theta)\end{Bmatrix}+\begin{Bmatrix}0\\ \mathbf{F}_0\end{Bmatrix} \tag{1}$$

where $\mathbf{Y}=\{y_V\ \theta_V\ y_1\ y_2\}^T$ is the vector of response of the vehicle. $\mathbf{M}_{V1}, \mathbf{M}_{V2}, \mathbf{C}_{V11}, \mathbf{C}_{V12}, \mathbf{C}_{V21}, \mathbf{C}_{V22}, \mathbf{K}_{V11}, \mathbf{K}_{V12}, \mathbf{K}_{V21}, \mathbf{K}_{V22}$ are the submatrices of the mass, damping and stiffness matrices of the vehicle, respectively, which are given in Appendix A. $\mathbf{F}_0$ is the static load vector of the vehicle. $\mathbf{F}(t,\theta) = \{F_1(t,\theta),\ldots,F_{N_F}(t,\theta)\}^T$ is the random bridge–vehicle interaction force vector, $N_F$ is the number of interaction forces, and $\theta$ denotes the random dimension. When $N_F = 2$, the interaction forces are

$$\begin{cases} F_1(t,\theta) = (m_1+a_2m_v)\,g + K_{t1}(y_1 - w(\hat{x}_1(t),t,\theta) \\ \quad -r(\hat{x}_1(t),\theta)) + C_{t1}(\dot{y}_1 - \dot{w}(\hat{x}_1(t),t,\theta) \\ \quad -w'(\hat{x}_1(t),t,\theta)\dot{\hat{x}}_1(t) - r'(\hat{x}_1(t),\theta)\dot{\hat{x}}_1(t)) \\ F_2(t,\theta) = (m_2+a_1m_v)\,g + K_{t2}(y_2 - w(\hat{x}_2(t),t,\theta) \\ \quad -r(\hat{x}_2(t),\theta)) + C_{t2}(\dot{y}_2 - \dot{w}(\hat{x}_2(t),t,\theta) \\ \quad -w'(\hat{x}_2(t),t,\theta)\dot{\hat{x}}_2(t) - r'(\hat{x}_2(t),\theta)\dot{\hat{x}}_2(t)) \end{cases} \tag{2}$$

where $K_{t1}, K_{t2}, C_{t1}, C_{t2}$ are the stiffness and damping of the two tires, respectively. The road surface roughness $r(x,\theta)$ is considered as a Gaussian random process with a Power Spectral Density (PSD) function given in Appendix B. $\hat{x}_1(t), \hat{x}_2(t)$ are the positions of the front and rear axles respectively on the bridge at time $t$. $g$ is the acceleration due to gravity. $w(\hat{x}_i(t),t,\theta)$ is the vertical random bridge displacement at the contact point of the $i$th interaction force at time $t$. The over-dot $(\cdot)$ denotes the differentiation with respect to time $t$ and the right prime $(')$ denotes the differentiation with respect to local coordinate $x$ in this paper.

The bridge is modeled as a planar simply supported Euler–Bernoulli beam. The mass density $\rho(x,\theta)$, Young's modulus $E(x,\theta)$ and damping $c(x,\theta)$ are assumed as non-Gaussian random processes, with mean value $\bar{\rho}, \bar{E}, \bar{c}$ and standard deviation $\sigma_\rho, \sigma_E, \sigma_c$, respectively, and the random parts of which are denoted as $\tilde{\rho}, \tilde{E}, \tilde{c}$, respectively. The equation of motion of the bridge structure can be written as

$$\rho(x,\theta)A\frac{\partial^2}{\partial t^2}w(x,t,\theta) + c(x,\theta)\frac{\partial}{\partial t}w(x,t,\theta) + E(x,\theta)I\frac{\partial^4}{\partial x^4}w(x,t,\theta) = \sum_{j=1}^{N_F} F_j(t,\theta)\delta(x - v_jt), \quad (j = 1,2,\ldots,N_F) \tag{3}$$

where $A$ and $I$ are the cross-sectional area and the moment of inertia of the beam, respectively, which are assumed as deterministic in this study. $w(x,t,\theta)$ is the random displacement which varies with the location $x$ and time $t$. $v_j$ is the constant speed of the $j$th moving interaction force $F_j(t,\theta)$; $\delta(t)$ is the Dirac delta function.

Employing the finite element method and with the assumption of Rayleigh damping, Eq. (3) will take the following form

$$\mathbf{M}_b\ddot{\mathbf{R}}(t,\theta) + \mathbf{C}_b\dot{\mathbf{R}}(t,\theta) + \mathbf{K}_b\mathbf{R}(t,\theta) = \mathbf{H}_b\mathbf{F}(t,\theta) \tag{4}$$

where $\mathbf{M}_b$, $\mathbf{C}_b$ and $\mathbf{K}_b$ are the random mass, damping and stiffness matrices, respectively, and $\mathbf{M}_b = \mathbf{M}_d + \tilde{\mathbf{M}}$, $\mathbf{C}_b = \mathbf{C}_d + \tilde{\mathbf{C}}$, $\mathbf{K}_b = \mathbf{K}_d + \tilde{\mathbf{K}}$. $\mathbf{M}_d, \tilde{\mathbf{M}}, \mathbf{C}_d, \tilde{\mathbf{C}}, \mathbf{K}_d, \tilde{\mathbf{K}}$ are the deterministic and random components of the mass, damping and stiffness matrices respectively of the beam and they can be generated by assembling the corresponding elemental matrices as

$$\mathbf{M}_d^e = \int_l \mathbf{H}^{eT}\bar{\rho}(x)A\mathbf{H}^e\mathrm{d}l \qquad \tilde{\mathbf{M}}^e = \int_l \mathbf{H}^{eT}\tilde{\rho}(x,\theta)A\mathbf{H}^e\mathrm{d}l$$
$$\mathbf{K}_d^e = \int_l \mathbf{B}^{eT}\bar{E}(x)I\mathbf{B}^e\mathrm{d}l \qquad \tilde{\mathbf{K}}^e = \int_l \mathbf{B}^{eT}\tilde{E}(x,\theta)I\mathbf{B}^e\mathrm{d}l \tag{5}$$

where $\mathbf{H}^e$ and $\mathbf{B}^e$ are the shape function matrix and strain–displacement matrix for each element, respectively. $l$ is the length of beam element. Rayleigh damping is assumed with the equation,

$$\mathbf{C}_b = c_M\mathbf{M}_b + c_K\mathbf{K}_b \tag{6}$$

where $c_M$ and $c_K$ are constants.