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Optimal sources in the MFS by minimizing a new merit function: Energy gap functional

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Abstract

The accuracy of the method of fundamental solutions (MFS) is strongly dependent on the distribution of source points. A proper choice of source points is still an important issue in the MFS. In this paper we derive a new merit function, namely the energy gap functional, whose minimum leads to the optimal distribution of source points. The new method can improve the accuracy of the MFS for solving the mixed boundary value problem as well as the Cauchy problem of the Laplace equation. The numerical tests confirm that the use of the optimal sources in the MFS performs well and the accuracy is increased.

Keywords: Laplace equation, Method of fundamental solutions (MFS), Cauchy problem, Energy gap functional, Optimal sources

1. Introduction

The method of fundamental solutions (MFS) is a truly meshless numerical method, popularly used in engineering computations. In the MFS the trial solution is a linear combination of fundamental solutions, which automatically satisfy the governing equation, and the expansion coefficients are uniquely determined from the specified boundary conditions. The MFS is easy to numerical implementation and has been used to solve the Laplace equation [1, 2].

However, the MFS has a serious defect that it may become highly ill-conditioned when the number of source points is increased or when the distances of source points are increased [3, 4]. For a better use of the MFS we need to reduce the condition number of the resulting linear equations system by searching a suitable distribution of source points. Various algorithms have been proposed to choose the optimal source points [5, 6, 7]. The leave-one-out cross validation algorithm has been developed by Chen et al. [8] to locate the source points, which leads to highly accurate result with a relatively low computational cost. In spite of these algorithms, a simple and truly optimal algorithm to determine the source points is still pending. In this paper we will develop an inherent energy gap functional of the Laplace equation to pick up the optimal sources.