Accepted Manuscript

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Kirill D. Nikitin, Kirill M. Terekhov, Yuri V. Vassilevski

PII: S0893-9659(18)30226-X
DOI: https://doi.org/10.1016/j.aml.2018.07.005
Reference: AML 5577

To appear in: *Applied Mathematics Letters*

Received date : 30 April 2018
Revised date : 2 July 2018
Accepted date : 2 July 2018

Please cite this article as: K.D. Nikitin, K.M. Terekhov, Yu.V. Vassilevski, Two methods of surface tension treatment in free surface flow simulations, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.07.005

TWO METHODS OF SURFACE TENSION TREATMENT IN FREE SURFACE FLOW SIMULATIONS *

KIRILL D. NIKITIN[†], KIRILL M. TEREKHOV[‡], AND YURI V. VASSILEVSKI[§]

Abstract. We describe our approach to treatment of surface tension in free surface flow simulations on adaptive octree-type grids. The approach is based on the semi-Lagrangian method for the transport and momentum equations and the pressure projection method to enforce the incompressibility constrain. The surface tension contributes to the Dirichlet boundary condition for the pressure equation at the projection step [18]. The treatment of surface tension is based either on accurate finite difference calculation of the mean curvature or on a curvature estimation by the implicit solution of conservative mean curvature flow problem [19]. The first method provides almost the second order accuracy in space for surface tension forces. The second method is characterised by greater stability and essentially larger time steps. Numerical experiments illustrate the main features of the methods.

Key words. free surface, incompressible Navier–Stokes equations, level-set method, semi-Lagrangian method, surface tension

1. Introduction. Accurate numerical treatment of surface tension remains the challenge for many years since it requires accurate evaluation of the free surface curvature. The accuracy of the numerical free surface approximation depends on the accuracy of numerical treatment of both viscous incompressible fluid flow and transport of the free surface. The height function method [15] was suggested for an accurate estimation of the curvature. According to Popinet [16], although high-order curvature estimation schemes have been introduced, the overall splitting schemes, present in the literature, are still formally first-order accurate.

Our splitting method involves accurate approximations of transport, eikonal, momentum, Poisson equations, as well as the second-order method for estimation of the free surface curvature. The numerically observed convergence rate of the resulting method demonstrates almost the second order. The momentum equation is solved with a novel combination of BDF2 (backward difference formula) and MacCormack predictor-corrector schemes. Advective terms within the scheme are treated with the semi-Lagrangian approach. Incompressibility condition is enforced with the projection method. The surface tension contributes to the Dirichlet boundary condition for the pressure equation at the projection step [18]. We consider the treatment of the surface tension by two methods: accurate finite difference calculation of the mean curvature, and curvature estimation by the implicit solution of a conservative mean curvature flow problem [19]. The first method provides almost the second order accuracy in space for surface tension forces. The second method allows us to relax considerably the time step restriction imposed by implicit and semi-implicit approaches [16].

The paper is organized as follows. Section 1 reviews the mathematical model and outlines the numerical methods used for the free surface flow modelling. Section 2 introduces two approaches for the accurate numerical treatment of the surface curvature. Section 3 presents the results of numerical experiments with an oscillating

*This work has been supported by the Russian Science Foundation through the grant 14-11-00434.

[†]Marchuk Institute of Numerical Mathematics, Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow; nikitin.kira@gmail.com

[‡]Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow; kirill.terehov@math.uh.edu

[§]Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow Institute of Physics and Technology, Sechenov University, Moscow; yuri.vassilevski@gmail.com