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Global nonexistence of solutions for von Karman equations with variable exponents

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ABSTRACT

We consider the von Karman equations with variable exponents:

$$u_{tt}+\Delta^2u+a|u_t|^{m(\cdot)-2}u_t=[u,F(u)]+b|u|^{p(\cdot)-2}u$$

where a and b are positive constants and the exponents $m(\cdot)$ and $p(\cdot)$ are given measurable functions. There are many literatures on the blow-up result of solutions for the wave equation. However, to the best of our knowledge, there is no blow-up result of solutions for von Karman equations. We investigate a finite time blow-up result of solutions with nonpositive initial energy as well as positive initial energy.

1. Introduction

Let Ω be a bounded domain in $\mathbb{R}^2$ with a sufficiently smooth boundary $\partial\Omega$. We consider the following von Karman equations:

$$u_{tt}+\Delta^2u+a|u_t|^{m(x)-2}u_t=[u,F(u)]+b|u|^{p(x)-2}u,\ \text{in } \Omega\times(0,\infty), \tag{1.1}$$

$$\Delta^2F(u)=-[u,u],\ \text{in } \Omega\times(0,\infty), \tag{1.2}$$

$$u=\frac{\partial u}{\partial\nu}=0, F(u)=\frac{\partial F(u)}{\partial\nu}=0,\ \text{on } \partial\Omega\times(0,\infty), \tag{1.3}$$

$$u(x,0)=u_0(x), u_t(x,0)=u_1(x),\ \text{in } \Omega, \tag{1.4}$$

where $a,b>0$, ν is the unit outward normal vector on $\partial\Omega$ and von Karman bracket is given by

$$[u,v]=u_{x_1x_1}v_{x_2x_2}+u_{x_2x_2}v_{x_1x_1}-2u_{x_1x_2}v_{x_1x_2}.$$

The exponents $m(x)$ and $p(x)$ are given measurable functions on Ω satisfying

$$2\le q_1\le q(x)\le q_2<\infty, \tag{1.5}$$

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with $q_1 := \operatorname{essinf}_{x\in\Omega} q(x)$, $q_2 := \operatorname{esssup}_{x\in\Omega} q(x)$, and the log-Hölder continuity condition for $A > 0$ and $0 < \delta < 1$:

$$|q(x) - q(y)| \leq -\frac{A}{\log|x-y|}, \text{ for a.e. } x, y \in \Omega, \text{ with } |x-y| < \delta. \tag{1.6}$$

When $a = b = 0$ in (1.1), Favini et al. [1] proved global existence, uniqueness and regularity of solutions for the equation with nonlinear boundary dissipation. Many researchers have studied the well-posedness, energy decay of solutions and existence of attractors for von Karman equations with dissipation (see e.g., [2–6] and reference therein). Cavalcanti et al. [7] proved the decay rate estimates for von Karman system with long memory. But there are few works on blow up of solutions for von Karman equations.

When $m(\cdot)$ and $p(\cdot)$ are constants, many authors have investigated the global existence, uniform decay rates and blow-up of solutions for the wave equation with nonlinear damping and source terms. For the more related works, we refer [8–14] and references therein.

In recent years, much attention has been paid to the research of mathematical models of parabolic, elliptic and hyperbolic equations with variable exponents. More details on these problems can be found in [15,16]. In fact, there are only few works regarding hyperbolic equations with variable exponents of nonlinearity. Messaoudi et al. [17] considered the existence and blow-up of solutions with negative initial energy for the nonlinear wave equation with variable exponents. Messaoudi and Talahmeh [18] studied the quasilinear wave equation with variable exponents. Motivated by previous works, we consider the blow up of solutions for the von Karman equations with variable exponents.

This paper is organized as follows: In Section 2, we recall the definition of the Lebesgue space with variable exponent $L^{p(\cdot)}(\Omega)$, as well as some of their properties. We also give some hypotheses and some necessary preliminaries. In Section 3, we state and prove a blow-up result of solutions for the problem (1.1)–(1.4) when the initial energy lies in positive as well as nonpositive.

2. Preliminaries

In this section, we give some preliminary facts about Lebesgue space with variable-exponents (see [19,20]). Let $p : \Omega \to [1, \infty]$ be a measurable function. We introduce the Lebesgue space with a variable exponent $p(\cdot)$

$$L^{p(\cdot)}(\Omega) := \{u : \Omega \to \mathbb{R};\ \text{measurable in } \Omega,\ \rho_{p(\cdot)}(\lambda u) < \infty,\ \text{for some } \lambda > 0\},\ \rho_{p(\cdot)}(u) = \int_\Omega |u|^{p(x)} dx.$$

Equipped with the following Luxembourg-type norm

$$\|u\|_{p(\cdot)} = \inf\left\{\lambda > 0 :\ \rho_{p(\cdot)}\left(\frac{u}{\lambda}\right) \leq 1\right\},$$

$L^{p(\cdot)}(\Omega)$ is a Banach space (see [20]). As usual, $(\cdot,\cdot)$ and $\|\cdot\|_p$ denote the inner product in the space $L^2(\Omega)$ and the norm of the space $L^p(\Omega)$, respectively. For brevity, we denote $\|\cdot\|_2$ by $\|\cdot\|$. If p satisfies condition (1.5), then the embedding $H_0^2(\Omega) \hookrightarrow L^{p(\cdot)}(\Omega)$ is continuous and compact. There exists the positive constant B satisfying

$$\|u\|_{p(\cdot)} \leq B\|\Delta u\|, \quad \text{for } u \in H_0^2(\Omega). \tag{2.1}$$

Lemma 2.1 *([20]). If p satisfies condition (1.5), then we have for any $u \in L^{p(\cdot)}(\Omega)$,*

$$\|u\|_{p(\cdot)} \leq 1 \text{ if and only if } \rho_{p(\cdot)}(u) \leq 1, \tag{2.2}$$

and

$$\min\{\|u\|_{p(\cdot)}^{p_1}, \|u\|_{p(\cdot)}^{p_2}\} \leq \rho_{p(\cdot)}(u) \leq \max\{\|u\|_{p(\cdot)}^{p_1}, \|u\|_{p(\cdot)}^{p_2}\}. \tag{2.3}$$