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NONDEGENERACY OF POSITIVE SOLUTIONS TO A KIRCHHOFF PROBLEM WITH CRITICAL SOBOLEV GROWTH

GONGBAO LI AND CHANG-LIN XIANG

ABSTRACT. In this paper, we prove uniqueness and nondegeneracy of positive solutions to the following Kirchhoff equations with critical growth

$$-\left(a + b \int_{\mathbb{R}^3} |\nabla u|^2\right) \Delta u = u^5, \quad u > 0 \quad \text{in } \mathbb{R}^3,$$

where $a, b > 0$ are positive constants. This result has potential applications in singular perturbation problems concerning Kirchhoff equations.

Keywords: Kirchhoff equations; Positive solutions; Uniqueness; Nondegeneracy

1. INTRODUCTION AND MAIN RESULT

In this paper, we are concerned about the nonlocal Kirchhoff type problem

$$-\left(a + b \int_{\mathbb{R}^3} |\nabla u|^2\right) \Delta u = u^5, \quad u > 0 \quad \text{in } \mathbb{R}^3, \quad (1.1)$$

where $a, b > 0$ are constants, $\Delta = \sum_{i=1}^3 \partial_{x_i x_i}$ is the usual Laplacian operator in \mathbb{R}^3 .

Denote by $D = D^{1,2}(\mathbb{R}^3)$ the completion of $C_0^\infty(\mathbb{R}^3)$ under the seminorm

$$\|\varphi\|_D^2 \equiv \int_{\mathbb{R}^3} |\nabla \varphi|^2.$$

A (weak) solution to Eq. (1.1) is a function $u \in D$ satisfying

$$\left(a + b \int_{\mathbb{R}^3} |\nabla u|^2\right) \int_{\mathbb{R}^3} \nabla u \cdot \nabla \varphi = \int_{\mathbb{R}^3} u^5 \varphi$$

for all $\varphi \in D$. By the Sobolev embedding $D \subset L^6(\mathbb{R}^3)$, all the integrals in the above equation are well defined.

Problem (1.1) and its variants have been studied extensively in the literature. Physician Kirchhoff [18] proposed the following time dependent wave equation

$$\rho \frac{\partial^2 u}{\partial t^2} - \left(\frac{P_0}{h} + \frac{E}{2L} \int_0^L \left| \frac{\partial u}{\partial x} \right|^2 \right) \frac{\partial^2 u}{\partial x^2} = 0$$

for the first time, in order to extend the classical D'Alembert's wave equations for free vibration of elastic strings. [5] and Pohozaev [25] contributed some early research on the study of Kirchhoff

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