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Gongbao Li, Chang-Lin Xiang

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NONDEGENERACY OF POSITIVE SOLUTIONS TO A KIRCHHOFF PROBLEM WITH CRITICAL SOBOLEV GROWTH

GONGBAO LI AND CHANG-LIN XIANG

ABSTRACT. In this paper, we prove uniqueness and nondegeneracy of positive solutions to the following Kirchhoff equations with critical growth

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\right)\Delta u=u^5,\quad u>0\quad \text{in } \mathbb{R}^3,$$

where $a,b>0$ are positive constants. This result has potential applications in singular perturbation problems concerning Kirchhoff equaitons.

Keywords: Kirchhoff equations; Positive solutions; Uniqueness; Nondegeneracy

1. Introduction and main result

In this paper, we are concerned about the nonlocal Kirchhoff type problem

$$-\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\right)\Delta u=u^5,\quad u>0\quad \text{in } \mathbb{R}^3, \tag{1.1}$$

where $a,b>0$ are constants, $\Delta=\sum_{i=1}^3\partial_{x_ix_i}$ is the usual Laplacian operator in $\mathbb{R}^3$.

Denote by $D=D^{1,2}(\mathbb{R}^3)$ the completion of $C_0^\infty(\mathbb{R}^3)$ under the seminorm

$$\|\varphi\|_D^2\equiv\int_{\mathbb{R}^3}|\nabla\varphi|^2.$$

A (weak) solution to Eq. (1.1) is a function $u\in D$ satisfying

$$\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\right)\int_{\mathbb{R}^3}\nabla u\cdot\nabla\varphi=\int_{\mathbb{R}^3}u^5\varphi$$

for all $\varphi\in D$. By the Sobolev embedding $D\subset L^6(\mathbb{R}^3)$, all the integrals in the above equation are well defined.

Problem (1.1) and its variants have been studied extensively in the literature. Physician Kirchhoff [18] proposed the following time dependent wave equation

$$\rho\frac{\partial^2u}{\partial t^2}-\left(\frac{P_0}{h}+\frac{E}{2L}\int_0^L\left|\frac{\partial u}{\partial x}\right|^2\right)\frac{\partial^2u}{\partial x^2}=0$$

for the first time, in order to extend the classical D'Alembert's wave equations for free vibration of elastic strings. [5] and Pohozaev [25] contributed some early research on the study of Kirchhoff

Corresponding author: Chang-Lin Xiang.

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