Accepted Manuscript

Continuation methods for time-periodic travelling-wave solutions to evolution equations

T.-S. Lin, D. Tseluiko, M.G. Blyth, S. Kalliadasis

PII: S0893-9659(18)30217-9
DOI: https://doi.org/10.1016/j.aml.2018.06.034
Reference: AML 5568

To appear in: *Applied Mathematics Letters*

Received date : 23 February 2018
Revised date : 28 June 2018
Accepted date : 28 June 2018

Continuation methods for time-periodic travelling-wave solutions to evolution equations

T.-S. Lin[c], D. Tseluiko[b], M. G. Blyth[a], S. Kalliadasis[d]

[a] *School of Mathematics, University of East Anglia, Norwich, NR4 7TJ, UK*
[b] *Department of Mathematical Sciences, Loughborough University, Loughborough, LE11 3TU, UK*
[c] *Department of Applied Mathematics, National Chiao Tung University, Hsinchu 30010, Taiwan*
[d] *Department of Chemical Engineering, Imperial College London, London, SW7 2AZ, UK*

Abstract

A numerical continuation method is developed to follow time-periodic travelling-wave solutions of both local and non-local evolution partial differential equations (PDEs). It is found that the equation for the speed of the moving coordinate can be derived naturally from the governing equations together with the condition that breaks the translational symmetry. The derived system of equations allows one to follow the branch of travelling-wave solutions as well as solutions that are time-periodic in a frame of reference travelling at a constant speed. Finally, we show as an example the bifurcation and stability analysis of single and double-pulse waves in long-wave models of electrified falling films.

Keywords: numerical continuation, evolution equation, long-wave model
2010 MSC: 00-01, 99-00

1. Introduction

We present a novel numerical continuation method which can be used to explore solutions of evolution equations with translational invariance. The method can be used to construct detailed bifurcation diagrams that include travelling wave solutions and their bound states as well as pulsating travelling wave solutions. Travelling waves are stationary in a frame travelling at constant speed. Pulsating travelling waves are time-periodic in a frame of reference travelling at an appropriate constant speed. An appropriate condition must be imposed to break the translational invariance. Normally there is also another symmetry associated with the fact that a new solution can be created from a given travelling wave solution by altering its volume. By treating the evolution equation in a moving frame as an infinite-dimensional dynamical system, steady states and time-periodic states can be obtained as fixed points and periodic orbits of this dynamical system. The speed of the moving frame is one of the unknowns and it can be eliminated by using the translational symmetry-breaking condition. Consequently we can exploit existing continuation methods for infinite-dimensional dynamical systems to explore the bifurcation structure for the evolution equation. A similar technique was developed by Lin *et al.* [1] to study the motion of a liquid film on the outside of a horizontal rotating cylinder; in this case there is no translational symmetry due to gravity. In the present work we apply our new continuation method to the particular case of a thin liquid film flowing down an inclined plane in the presence of an electric field.

The description of the method is presented in Sec. 2 and in Sec. 3 we present our results including detailed bifurcation diagrams and time-periodic solutions. Finally in Sec. 4 we summarise our findings.

2. Numerical continuation of solutions of evolution equations

Consider the general one-dimensional evolution equation

$$\partial_t h = \partial_x (F[h]), \qquad (1)$$