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Dynamics of the breathers and rogue waves in the higher-order nonlinear Schrödinger equation☆

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ABSTRACT

In this paper, the higher-order nonlinear Schrödinger equation, which can be widely used to describe the dynamics of the ultrashort pulses in optical fibers, is under investigation. By means of the modified Darboux transformation, the hierarchies of breather wave and rogue wave solutions are generated from the trivial solution. Furthermore, the main characteristics of the breather and rogue waves are graphically discussed. The results show that the extreme behavior of the breather wave yields the rogue wave for the higher-order nonlinear Schrödinger equation.

1. Introduction

As we well know rogue waves (RWs) (alias freak waves, monster waves and abnormal waves etc.) are notorious for causing disastrous consequences in the ocean [1], and can appear in the shallow waters (or the deep ocean) [2,3]. A remarkable feature of the wave is that they "suddenly come from nowhere and disappear without a trace as the time evolves", and it only has a very short period of time before they attack a ship. Lately, RWs have drawn more and more experimental and theoretical attention in some related fields such as plasmas physics, optical fibers, Bose–Einstein condensates (BECs), finance and other fields [4–12]. The first mathematical model for describing RWs is the focusing nonlinear Schrödinger (FNLS) equation [13–16]

$$iu_t + u_{xx} + 2u|u|^2 = 0, \tag{1.1}$$

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which can arise in many physical settings and widely describe the phenomenon of envelope solitons [17,18]. Recently, by employing Darboux transformation (DT) and Hirota's bilinear (HB) method etc., [19–33], there have been a number of studies to investigate exact solutions of other systems.

The nonlinear Schrödinger (NLS) equation is well-known to be an important integrable equation in mathematical physics. There are many physical contexts where the NLS equation often appears. For instance, the NLS equation can be widely used to describe the weakly nonlinear surface wave in deep water. Besides, the NLS equation can model the soliton propagation in optical fibers where only the group velocity dispersion and the self-phase modulation effects are considered. However, for ultrashort pulse in optical fibers, the effects of the higher-order dispersion, the self-steepening and the stimulated Raman scattering should be taken into account. Nevertheless, it has been shown recently that all the higher-order terms are directly connected to the generalized NLS equation that is widely applied for ultrashort pulse propagation in optical fibers [34,35]. Therefore, in this paper, we mainly focus on the higher-order nonlinear Schrödinger (HNLS) equation [36]

$$iq_x + \alpha_2 K_2(q) - i\alpha_3 K_3(q) + \alpha_4 K_4(q) - i\alpha_5 K_5(q) = 0, \tag{1.2}$$

where

$$\begin{cases} K_2 = q_{tt} + 2q|q|^2, \\ K_3 = q_{ttt} + 6q_t|q|^2, \\ K_4 = q_{tttt} + 8|q|^2 q_{tt} + 6|q|^4 q + 4|q_t|^2 q + 6\bar{q}q_t^2 + 2q^2\bar{q}_{tt}, \\ K_5 = q_{ttttt} + 10|q|^2 q_{ttt} + 10\left(q|q_t|^2\right)_t + 20\bar{q}q_t q_{tt} + 30|q|^4 q_t, \end{cases} \tag{1.3}$$

with $u = u(x,t)$ is a complex-valued function and $\alpha_i (i = 2,3,4,5)$ are all real constants. Taking $\alpha_2 = 1$ and $\alpha_3 = \alpha_4 = \alpha_5 = 0$ in Eq.(1.2), Eq.(1.2) can be reduced to the standard NLS equation (1.1). Then by restricting $\alpha_2 = 1$ and $\alpha_3 = \alpha_4 = \alpha_5 = 0$, the rogue wave and breather wave solutions of the HNLS equation (1.2) can be reduced to those of the standard NLS equation (1.1).

Many mathematical physicists have studied the particular cases of Eq. (1.2). To the best of authors' knowledge, there are very few studies on (1.2). Based on symbolic calculation methods [37–48], The primary purpose of the present article is to employ a direct method (i.e., modified Darboux transformation (mDT)) to construct the breather and rogue wave solutions of Eq. (1.2). Additionally, the dynamic behaviors of the solutions are also considered by choosing appropriate parameters.

The structure of this paper is given as below. In Section 2, the breather wave solutions of Eq. (1.2) are obtained by using mDT. In Section 3, the Taylor expansion is used to explicitly derive the rogue wave solutions of Eq. (1.2). Finally, some conclusions and discussions are provided.

2. Breather wave solutions

Based on the results in [36], we take the trivial solution as a plane wave $u_0 = \exp(2(\alpha_2 + 3\alpha_4))$ and restrict λ as purely imaginary. The general form of the first-order breather solution reads

$$q_{\mathrm{bw}}^{[1]} = \left[\frac{k^3\cosh(Vx) + 2i\delta k\sinh(Vx)}{2k\cosh(Vx) - \delta\cos(k(t + V_b x))} - 1\right]\exp(\omega x), \tag{2.1}$$

where

$$V = 2\delta\left[\alpha_2 - \alpha_4\left(k^2 - 6\right)\right], \quad \omega = 2(\alpha_2 + 3\alpha_4),$$
$$V_b = \alpha_5\left(k^4 - 10k^2 + 30\right), \quad \delta = \frac{k\sqrt{4 - k^2}}{2}. \tag{2.2}$$