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A BOUNDARY LAYER APPROACH TO CREEPING WAVES

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Abstract

We consider the eigenvalue problem for the Laplace operator in a two dimensional domain exterior to a smooth, closed convex curve C, on which the eigenfunctions are to vanish. Waves whose wavelengths λ are very small may be viewed as particles traveling along specific paths termed rays, along which the waves propagate. Small wavelengths λ correspond to large wave numbers $k = \frac{2\pi}{\lambda}$, which are the eigenvalues. Therefore, we restrict attention to the consideration of large eigenvalues. If the amplitude of the eigenfunctions is appreciable only in a thin region attached to the boundary and is negligibly small beyond the layer, they correspond to creeping waves. We employ a boundary layer approach to the problem.

Introduction.

Waves whose wavelengths λ are very small may be viewed as particles traveling along specific paths termed rays, along which the waves propagate. According to the Geometrical Theory of Optics (GTO), waves are interpreted in terms of rays, with only incident and reflected rays considered. Sommerfeld and Runge [1] showed that GTO is the leading term in an asymptotic series expansion for wavenumber $k = \frac{2\pi}{\lambda} \gg 1$, though not as formally, nor as completely as is now understood. Consider light waves impinging on a smooth convex solid object. In the region behind the solid, termed the shadow region, no light should be seen. However, this contradicts the observation of light in the shadow region. This is due to the fact that certain waves were not accounted for in GTO. These are diffracted waves. Keller accounted for these and other diffracted waves in his Geometrical Theory of Diffraction (GTD) [2], showing that diffracted waves can also be described as asymptotic approximations in the small wavelength limit. According to this theory, when incident rays are tangent to the solid surface, they lead to rays that travel along the surface (creeping waves), and shed waves into the exterior of the surface in the direction of the tangent, thus decreasing in strength as they propagate. These include rays launched into the shadow region, thus accounting for the appearance of light there. The problem was worked on by many, including Walter Franz [3], who coined the term "creeping wave". Later it was shown that the same phenomenon occurs for acoustic waves [4], leading to a scalar equation, rather than a vector equation describing light waves. It also arises in solid mechanics, and other wave systems, and has been used successfully in nondestructive testing and in seismology, e.g., in the study of earthquakes, as well as in other applications.

There are many types of wave modes. The amplitudes of the creeping wave modes are appreciable only in a thin layer near the boundary, but negligible away from the boundary. Our goal is to describe such modes. They are analogous to the related problem of the whispering gallery waves, in which a person speaking near the enclosing wall of a room can be heard across the room near the wall, but not in the center of the room. Such a wave is a diffracted wave. Ray theory for diffracted waves was introduced by Keller in his Geometrical Theory of Diffraction (GTD). To determine what combination of modes is the solution of a particular problem, it is necessary to