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An efficient ergodic simulation of multivariate stochastic processes with spectral representation

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ABSTRACT

A simulation formula to generate stationary multivariate stochastic processes is derived from the Fourier–Stieltjes integral of spectral representation. It is proved that the proposed algorithm generates ergodic sample functions in the mean value and in the correlation when the sample length is equal to one period (the generated sample functions are periodic). The algorithm is very efficient computationally since it takes advantage of the fast Fourier transform technique. The simulation of longitudinal wind velocity fluctuations and the simulation of longitudinal and vertical wind fluctuating components on a bridge deck are performed. It has been noted that there are good agreements between the temporal and target auto-/cross-correlation functions of simulated wind velocities.

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1. Introduction

The time-domain approach based on Monte Carlo methodology appears be very suitable for the solution of certain problems in stochastic mechanics involving nonlinearity, stochastic stability, parametric excitation, etc. One of the most important steps of a Monte Carlo time-domain simulation is to generate the sample functions of the stochastic processes, fields, or waves in the problem.

Although there are now many methods available to simulate such multivariate stochastic fields [1], spectral representation methods appear to be most widely used because of their versatility and robustness, and they are discussed in this paper. The basic method for analyzing a one-dimensional, one-variate Gaussian process appeared in the 1950s. To our knowledge, Shinozuka [2,3] was the foremost person who applied the spectral representation method for simulation purposes including multidimensional, multivariate, and nonstationary cases. Yang [4] showed that the fast Fourier transform (FFT) technique could be used to improve the computational efficiency of the spectral representation algorithm dramatically, and proposed a formula to simulate random envelope processes. Shinozuka [5] extended the application of the FFT technique to multidimensional cases. In 1996, Deodatis [6] further extended the spectral representation method to generate ergodic sample functions of multivariate stochastic processes. Moreover, several review papers on the subject of simulation using the spectral representation method were written by Shinozuka [7], and Shinozuka and Deodatis [8].

The spectral representation method has been further developed in the actual application of simulation of wind fields, earthquake waves, etc. [9–17]. Through introducing an explicit form of the Cholesky decomposition of a special power spectrum density (PSD) matrix, Yang [13] and Cao and Xiang [14] greatly improved the efficiency of Shinozuka's and Deodatis' methods respectively for simulating the wind velocity along the horizontal axis of bridge decks. Although the Deodatis method can produce unconditionally stable and satisfactory results, it is computationally expensive due to the repetitive decomposition of the power density matrix when a number of random processes are to be simulated. Meanwhile, the improvement to the Deodatis method made by Cao and Xiang [14] has a severe restriction on the simulated stochastic field: both the auto-spectral power spectra at simulated points and their spacing must be identical.

In this paper, an efficient simulation formula to generate stationary multivariate stochastic processes is derived from the Fourier–Stieltjes integral of spectral representation. The proposed algorithm generates ergodic sample functions in the mean value and in the correlation when the sample length is equal to one period. The algorithm is very efficient computationally since it takes advantage of the fast Fourier transform technique. Moreover, the simulation of longitudinal wind velocity fluctuations and the simulation of longitudinal and vertical wind fluctuating components on a horizontal bridge deck are performed in order to demonstrate the capability and efficiency of the proposed algorithm.

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2. Simulation formula

Consider a set of n stationary stochastic processes $\{X_j^0(t)\}$ ($j = 1, 2, \ldots, n$) with their mean values being zero, where the superscript 0 denotes the target function. The cross-correlation matrix $\mathbf{R}^0(\tau)$ is given by

$$\mathbf{R}^0(\tau) = \begin{bmatrix} R_{11}^0(\tau) & R_{12}^0(\tau) & \cdots & R_{1n}^0(\tau) \\ R_{21}^0(\tau) & R_{22}^0(\tau) & \cdots & R_{2n}^0(\tau) \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1}^0(\tau) & R_{n2}^0(\tau) & \cdots & R_{nn}^0(\tau) \end{bmatrix} \quad (1)$$

and the two-sided cross-spectral density matrix $\mathbf{S}^0(\omega)$ is given by

$$\mathbf{S}^0(\omega) = \begin{bmatrix} S_{11}^0(\omega) & S_{12}^0(\omega) & \cdots & S_{1n}^0(\omega) \\ S_{21}^0(\omega) & S_{22}^0(\omega) & \cdots & S_{2n}^0(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1}^0(\omega) & S_{n2}^0(\omega) & \cdots & S_{nn}^0(\omega) \end{bmatrix}. \quad (2)$$

The elements of the cross-correlation matrix are related to the corresponding elements of the cross-spectral density matrix through the Wiener–Khintchine transformation

$$S_{jk}^0(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{jk}^0(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau, \quad j, k = 1, 2, \ldots, n \quad (3a)$$

$$R_{jk}^0(\tau) = \int_{-\infty}^{\infty} S_{jk}^0(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega, \quad j, k = 1, 2, \ldots, n. \quad (3b)$$

For actual stochastic processes, the auto-spectral density function is a real and nonnegative function of ω and the cross-spectral density function is a generally complex function of ω. The following relations are valid:

$$\begin{aligned} &S_{jj}^0(\omega) = S_{jj}^0(-\omega), \qquad S_{jk}^0(\omega) = S_{jk}^{0*}(-\omega), \\ &S_{jk}^0(\omega) = S_{kj}^{0*}(\omega) \end{aligned} , \quad (4)$$

where $j, k = 1, 2, \ldots, n$, and the asterisk denotes the complex conjugate. Thus the cross-spectral density matrix $\mathbf{S}^0(\omega)$ is Hermitian. When the simulated stochastic processes are independent, $\mathbf{S}^0(\omega)$ is usually nonsingular. The matrix $\mathbf{S}^0(\omega)$ can be decomposed into the following format:

$$\mathbf{S}^0(\omega) = \mathbf{H}^*(\omega)\mathbf{H}^T(\omega) \quad (5)$$

$$\mathbf{H}(\omega) = \begin{bmatrix} H_{11}(\omega) & 0 & \cdots & 0 \\ H_{21}(\omega) & H_{22}(\omega) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ H_{n1}(\omega) & H_{n2}(\omega) & \cdots & H_{nn}(\omega) \end{bmatrix},$$

where the superscript T denotes the transpose of a matrix; $\mathbf{H}(\omega)$ is the lower triangular matrix. This decomposition can be performed using Cholesky's method. In general, the diagonal and off-diagonal elements of the lower triangular matrix are complex functions of ω. There are the following relations for the elements of matrix $\mathbf{H}(\omega)$:

$$H_{jk}(\omega) = H_{jk}^*(-\omega), \quad j, k = 1, 2, \ldots, n, j \geq k. \quad (6)$$

If the elements $H_{jk}(\omega)$ are written in polar form as

$$H_{jk}(\omega) = |H_{jk}(\omega)|\mathrm{e}^{\mathrm{i}\theta_{jk}(\omega)}, \quad j, k = 1, 2, \ldots, n, \quad (7)$$

where $\theta_{jk}(\omega)$ is the complex angle of $H_{jk}(\omega)$ and is given by

$$\theta_{jk}(\omega) = \tan^{-1}\left\{\frac{\mathrm{Im}\,[H_{jk}(\omega)]}{\mathrm{Re}\,[H_{jk}(\omega)]}\right\} \quad (8)$$

with Im[ ] and Re[ ] being the imaginary and real parts of the complex function in parentheses, respectively, then Eq. (6) is written equivalently as

$$|H_{jk}(\omega)| = |H_{jk}(-\omega)|, \quad j, k = 1, 2, \ldots, n, j \geq k \quad (9a)$$

$$\theta_{jk}(\omega) = -\theta_{jk}(-\omega), \quad j, k = 1, 2, \ldots, n, j \geq k. \quad (9b)$$

Based on the spectral analysis, each of the n stochastic processes may be expressed as a Fourier–Stieltjes integral over a random Fourier increment [18]:

$$X_j(t) = \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega), \quad j = 1, 2, \ldots, n. \quad (10)$$

The random increment must satisfy the following orthogonality conditions:

$$\begin{aligned} &E[\mathrm{d}Z_j(\omega)] = 0 \\ &E[\mathrm{d}Z_j^*(\omega)\mathrm{d}Z_k(\omega')] = 0, \quad \omega \neq \omega' \\ &E[\mathrm{d}Z_j^*(\omega)\mathrm{d}Z_k(\omega)] = S_{jk}(\omega)\mathrm{d}\omega, \end{aligned} \quad (11)$$

where $E[\cdot]$ is the mathematical expectation. It is noted that the two Fourier increments $\mathrm{d}Z_j(\omega)$ and $\mathrm{d}Z_k(\omega')$ are statistically correlated only when $\omega = \omega'$. When the simulated processes $X_j(t)$ ($j = 1, 2, \ldots, n$) are real, the relation $\mathrm{d}Z_j(-\omega) = \mathrm{d}Z_j^*(\omega)$ is also required.

Eq. (10) can be rewritten as

$$\begin{aligned} X_j(t) &= \int_0^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega) + \int_{-\infty}^{0} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega) \\ &= \int_0^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega) + \int_0^{\infty} \mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}Z_j(-\omega). \end{aligned} \quad (12)$$

Introducing the relations $\mathrm{e}^{-\mathrm{i}\omega t} = (\mathrm{e}^{\mathrm{i}\omega t})^*$ and $\mathrm{d}Z_j(-\omega) = \mathrm{d}Z_j^*(\omega)$, then

$$\begin{aligned} X_j(t) &= \int_0^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega) + \int_0^{\infty} [\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega)]^* \\ &= 2\mathrm{Re}\left[\int_0^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega)\right]. \end{aligned} \quad (13)$$

The discrete approximation to the random Fourier increment in Eq. (13) can be constructed in various ways. In the proposed scheme, the following approximate expression is utilized:

$$\mathrm{d}Z_j(\omega)|_{\omega=\omega_l} \approx \Delta Z_j(\omega_l) = \sum_{m=1}^{n} H_{jm}(\omega_l)\mathrm{e}^{\mathrm{i}\phi_{ml}}\sqrt{\Delta\omega} \quad (14)$$

$$\omega_l = l\Delta\omega + \Delta\omega/2, \quad l = 0, \ldots, N-1, \quad (15)$$

where N is a sufficiently large number; $\Delta\omega = \omega_{up}/N$ is the frequency increment, ω_{up} is the upper cutoff frequency, with the condition that, when $\omega > \omega_{up}$, the value of $\mathbf{S}^0(\omega)$ is trivial; ϕ_{ml} are sequences of independent random phase angles, uniformly distributed over the interval $[0, 2\pi]$. Meanwhile, the exponential term in Eq. (13) is discretized using the double-indexing frequency [6]

$$\begin{aligned} &\mathrm{e}^{\mathrm{i}\omega t}|_{\omega=\omega_{ml}} = \mathrm{e}^{\mathrm{i}\omega_{ml}t}, \qquad \omega_{ml} = l\Delta\omega + \frac{m}{n}\Delta\omega, \\ &l = 0, 1, \ldots, N-1. \end{aligned} \quad (16)$$

Inserting Eqs. (14), (16) and (7) into Eq. (13), then

$$\begin{aligned} X_j(t) &= \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z_j(\omega) \\ &\approx 2\mathrm{Re}\left[\sum_{l=0}^{N-1}\sum_{m=1}^{n} H_{jm}(\omega_l)\mathrm{e}^{\mathrm{i}(\omega_{ml}t+\phi_{ml})}\sqrt{\Delta\omega}\right] \end{aligned}$$

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