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Classification of global behavior of a system of rational difference equations

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Abstract: This paper deals with a system of rational difference equations

$$x_{n+1} = \frac{ay_n + b}{cy_n + d}, \quad y_{n+1} = \frac{ax_n + b}{cx_n + d}, \quad n = 0, 1, 2, \ldots,$$

where a, b, c, d are real numbers with $c \neq 0$ and $ad - bc \neq 0$. We establish a representation formula of solutions of the system and classify global behavior of solutions when no initial values belong to the forbidden set of the system.

Keywords: Rational difference system; Stability; Periodicity; Forbidden set.

2010 Mathematics Subject Classifications: 39A10; 39A23; 39A30.

1. Introduction

We consider a system of rational difference equations

$$x_{n+1} = \frac{ay_n + b}{cy_n + d}, \quad y_{n+1} = \frac{ax_n + b}{cx_n + d}, \quad n = 0, 1, 2, \ldots, \tag{1}$$

where parameters a, b, c, d and initial values x_0, y_0 are real numbers. To avoid degenerate cases, we will assume throughout this paper that

$$c \neq 0 \quad \text{and} \quad ad - bc \neq 0. \tag{2}$$

Indeed, when $c = 0$, system (1) is a linear system. Also, when $c \neq 0$ and $ad - bc = 0$, system (1) is reduced to the trivial relation $x_n = y_n = a/c$ for $n = 1, 2, \ldots$.

Let $\{(x_n, y_n)\}_{n=0}^{\infty}$ be a solution of (1). In case $x_0 = y_0$, we observe that $x_n = y_n$ for $n = 1, 2, \ldots$ and dynamical behavior of (1) coincides with that of a scalar Riccati difference equation

$$x_{n+1} = \frac{ax_n + b}{cx_n + d}, \quad n = 0, 1, 2, \ldots. \tag{3}$$

By the substitution $x_n = z_{n+1}/z_n - d/c$, equation (3) is reduced to the form

$$z_{n+2} - \frac{a + d}{c} z_{n+1} + \frac{ad - bc}{c^2} z_n = 0. \tag{4}$$

In 1955, Brand [1] obtained a classification of global behavior of solutions of (3) by the study of (4) in detail; see also [2, 7, 8].

In case $a = 0$ and $b = c = d = 1$, equation (3) becomes

$$x_{n+1} = \frac{1}{x_n + 1}, \quad n = 0, 1, 2, \ldots. \tag{5}$$

In 2013, Tollu et al. [9] gave the following representation formula of solutions of (5) by using Fibonacci numbers. Here the Fibonacci sequence $\{F_n\}_{n=0}^{\infty}$ is defined by

$$F_{n+2} = F_{n+1} + F_n, \quad n = 0, 1, 2, \ldots$$

with $F_0 = 0$ and $F_1 = 1$. For brevity, we put $e_+ = (-1 + \sqrt{5})/2$, $e_- = (-1 - \sqrt{5})/2$.

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