Accepted Manuscript

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PII: S0893-9659(18)30160-5
DOI: https://doi.org/10.1016/j.aml.2018.05.018
Reference: AML 5525

To appear in: *Applied Mathematics Letters*

Received date : 23 April 2018
Revised date : 22 May 2018
Accepted date : 22 May 2018

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Abstract

In the present work, we deal with the logistic equation and its stability with respect to perturbations. In fact, for perturbations below a certain threshold, we provide an estimate for the difference between solutions of the exact and perturbed models, which scales linearly with the magnitude of the perturbation. This actually proves the conditional Ulam stability of the logistic equation.

Keywords: Logistic model, approximate solution, Ulam stability

1. Introduction

The logistic growth model, proposed by P. F. Verhulst in the first half of the 19^{th} century, represents not only a cornerstone in the study of population dynamics and mathematical biology, but is also a typical example of a nonlinear dynamical system.

The governing equation

$$\frac{dP}{d\tau} = rP\left(1 - \frac{P}{K}\right) \tag{1}$$

describes the time evolution of the size P of a population whose growth obeys a self-limiting version, which accounts for the scarcity of available resources, of the Malthusian law. The per capita growth rate $r\left(1 - \frac{P}{K}\right)$ decreases linearly as a function of the population size and becomes zero as P reaches the maximum carrying capacity $K > 0$ of the environment.

In what follows, we will be concerned mainly with the most commonly used, dimensionless form of the logistic model

$$\frac{dy}{dt} = y - y^2, \tag{2}$$

which arises after passing to the rescaled variables $t = r\tau$ and $y = P/K$ in (1). More precisely, we are interested in the *Ulam stability* of (2) (cf. [14] or [3]) in the sense that there exists $C > 0$ such that for any approximate solution

$$\widetilde{y} \in C^1[0,\infty) \quad \text{with} \quad \left|\frac{d\widetilde{y}}{dt} - \widetilde{y} + \widetilde{y}^2\right| \le \varepsilon,$$

there exists a nearby exact solution $y \in C^1[0,\infty)$ of (2) such that

$$|\widetilde{y}(t) - y(t)| \le C\varepsilon \quad \text{for all} \quad t \ge 0.$$

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