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Exponential ultimate boundedness of impulsive stochastic delay differential equations☆

Liguang Xu[a], Zhenlei Dai[a], Danhua He[b,*]

[a] *Department of Applied Mathematics, Zhejiang University of Technology, Hangzhou 310023, PR China*
[b] *Department of Mathematics, Zhejiang International Studies University, Hangzhou 310023, PR China*

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ABSTRACT

This paper is devoted to the investigation of the ultimate boundedness of impulsive stochastic delay differential equations. The pth moment global exponential ultimate boundedness criteria are obtained based on the Itô formula and the successful construction of suitable Lyapunov functionals, and the estimated exponential convergence rate and the ultimate bound are provided as well. It is shown that unbounded stochastic delay differential equations can be turned into bounded ones by impulses.

1. Introduction

Impulsive stochastic differential equations are an important type of hybrid dynamical systems, which have the characteristics of both impulsive differential equations and stochastic differential equations. As theory developed and perfected in impulsive differential equations and stochastic differential equations, more and more scholars started turning their eyes towards the theory of impulsive stochastic differential equations. Many interesting contributions are now available in the literature concerning stability [1,2], convergence [3], controllability [4], boundedness [5,6], and existence and uniqueness [7,8] of impulsive stochastic differential equations.

On the other hand, time delays occur frequently in many practical engineering systems, which are usually the source of oscillation, instability and poor performance of the systems. Thus it is necessary to study the impulsive stochastic differential equations with time delays. Recently, great efforts have been devoted to extend the qualitative and stability theory from the delay-free impulsive stochastic differential equations to the delay ones [9–14]. But unfortunately, the results that the unbounded stochastic delay differential equations can be turned into bounded ones by impulses are hard to obtain.

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* Corresponding author.
E-mail addresses: xlg132@126.com (L. Xu), zldai720@163.com (Z. Dai), danhuahe@126.com (D. He).

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The above discussion leaves us with the main question: how to obtain the results which imply that impulses can contribute to the ultimate boundedness for stochastic delay differential equations even if they are unbounded themselves? This issue constitutes the main motivation of this paper. Based on the Itô formula and the successful construction of suitable Lyapunov functionals, the pth moment global exponential ultimate boundedness criteria are obtained and the estimated exponential convergence rate (ECR) and the ultimate bound are provided as well. The finding shows that impulses can contribute to the ultimate boundedness for stochastic delay differential equations even if they are unbounded themselves.

2. Preliminaries

Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space with the norm $|\cdot|$, $\mathbb{R}_{t_0} = [t_0, \infty)$ and $\mathbb{N} = \{1, 2, 3, \ldots\}$. Let $\mathcal{L}$ denote the well-known $\mathcal{L}$-operator given by the Itô formula (cf. [15]). Let $PC[[-\tau, 0], \mathbb{R}^n]$ denote the family of all piecewise right continuous functions $\phi : [-\tau, 0] \to \mathbb{R}^n$ with the norm $\|\phi\| = \sup_{-\tau\le s\le 0}|\phi(s)|$. The symbols $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ stand for the maximum and the minimum eigenvalue of the corresponding matrix, respectively. Let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\ge0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\ge t_0}$ satisfying the usual conditions, and $\omega(t) = (\omega_1(t), \ldots, \omega_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. Let $PC^b_{\mathscr{F}_t}[[-\tau, 0], \mathbb{R}^n]$ denote the family of all bounded $\mathscr{F}_t$-measurable, $PC[[-\tau, 0], \mathbb{R}^n]$-valued random variables ϕ, satisfying $\|\phi\|^p_{L^p} = \mathbb{E}[\sup_{-\tau\le\theta\le0}|\phi(\theta)|^p] < \infty$, where $\mathbb{E}$ denotes the expectation of stochastic process.

Consider the following impulsive stochastic delay differential equation:

$$\begin{cases} dx(t) = f(t, x(t), x(t-\tau))dt + \sigma(t, x(t), x(t-\tau))d\omega(t),\ t \ne t_k, t \ge t_0, \\ \Delta x(t_k) = I_k(x(t_k^-), t_k),\ k \in \mathbb{N}, \\ x(t_0 + s) = \phi(s), s \in [-\tau, 0], \end{cases} \tag{1}$$

where the initial function $\phi(s) \in PC^b_{\mathscr{F}_t}[[-\tau, 0], \mathbb{R}^n]$, $f : \mathbb{R}_{t_0} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : \mathbb{R}_{t_0} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$, $I_k : \mathbb{R}^n \times \mathbb{R}_{t_0} \to \mathbb{R}^n$, and the fixed impulsive moments $\{t_k, k \in \mathbb{N}\}$ satisfy $0 \le t_0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. The symbol $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ where $x(t_k^+) = \lim_{\varepsilon\to0^+} x(t_k + \varepsilon)$ and $x(t_k^-) = \lim_{\varepsilon\to0^-} x(t_k + \varepsilon)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively.

Throughout this paper, we assume that for any initial function $\phi(s) \in PC^b_{\mathscr{F}_t}[[-\tau, 0], \mathbb{R}^n]$, there exists at least one solution of Eq. (1), which is left continuous at each t_k, i.e. $x(t_k^-) = x(t_k)$. For results on the existence and uniqueness of the solution of impulsive stochastic delay differential equations, the reader is referred to [16].

Definition 2.1. Eq. (1) is said to be pth moment globally exponentially ultimately bounded (p-GEUB) with the ECR λ and the ultimate bound M if there exist two positive constants λ, K and one nonnegative constant M such that for any initial condition $\phi \in PC^b_{\mathscr{F}_t}[[-\tau, 0], \mathbb{R}^n]$, $\mathbb{E}\|x(t)\|^p \le K\, \|\phi\|^p_{L^p}\, e^{-\lambda(t-t_0)} + M,\ p > 0, t \ge t_0$. Especially, Eq. (1) is said to be pth moment globally exponentially stable (p-GES) with the ECR λ when $M = 0$. When $p = 2$, p-GEUB and p-GES are usually called to be globally exponentially ultimately bounded in mean square (MS-GEUB) and globally exponentially stable in mean square (MS-GES), respectively.

Lemma 2.1. *[17]. For $y_i \ge 0$, $h_i > 0$ and $\sum_{i=1}^n h_i = 1$, $\Pi_{i=1}^n y_i^{h_i} \le \sum_{i=1}^n h_i y_i$.*

3. Main results

Theorem 3.1. *Suppose that there exist constants $C_1 \in \mathbb{R}$, $C_2 \ge 0$, $C_3 \ge 0$, $C_4 \ge 0$, $\Upsilon_1 \ge 0$, $\Upsilon_2 \ge 0$, $\zeta > 1$, $\vartheta > 0$ $\rho \ge 0$, $\beta_k > 0$, $\xi > 1$ and a symmetric positive-definite matrix $\mathcal{P}$ such that*