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A result on a non-autonomous Kirchhoff type equation involving critical term*

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Abstract: By using the method of Nehari manifold, we obtain the existence of a positive ground state solution for the following non-autonomous Kirchhoff type equation

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx\right)\triangle u+V(x)u=u^{\frac{N+2}{N-2}}, & x\in\mathbb{R}^N,\\ u\in D^{1,2}(\mathbb{R}^N),\ N=3,4, \end{cases}$$

where $a>0, b>0$ and V satisfies some appropriate assumptions.

Key words: Kirchhoff type equation; Nehari manifold; Positive solution

Mathematics Subject Classification Numbers: 35J60; 35B09; 35B38

1 Introduction and main result

In this paper, we study the existence of a positive solution to the equation

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx\right)\triangle u+V(x)u=u^{\frac{N+2}{N-2}}, & x\in\mathbb{R}^N,\\ u\in D^{1,2}(\mathbb{R}^N),\ N=3,4, \end{cases} \tag{1.1}$$

where $a>0, b>0$ and $V\in L^{\frac{N}{2}}(\mathbb{R}^N)$.

Kirchhoff type equation arises in the description of nonlinear vibrations of an elastic string (see [4]) and has been studied extensively by many authors, for example, Azzollini [1, 2], Li and Liao [5], Liu et al. [6], Tang and Chen [7], Xie et al. [9] for related results.

Specially, motivated by Benci and Cerami [3], Xie et al. [9] studied the bound state solution for equation (1.1) with $N=3$ and they assumed that potential function $V\in L^{\frac{3}{2}}(\mathbb{R}^3)$ is a nonnegative function. But in the present paper we investigate the ground state solution of equation (1.1), where $N=3,4$ and V as a potential function can be negative or sign-changing. As we know, our situation has not been discussed.

When $V(x)\equiv 0$, equation (1.1) reduces to the following equation

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx\right)\triangle u=u^{\frac{N+2}{N-2}}, & x\in\mathbb{R}^N,\\ u\in D^{1,2}(\mathbb{R}^N),\ N=3,4. \end{cases} \tag{1.2}$$

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