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Chun Li

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Solutions with minimal period for non-autonomous second-order Hamiltonian systems☆

Chun Li

School of Mathematics and Statistics, Southwest University, Chongqing 400715, China

Abstract

In this paper, we study the existence of subharmonic solutions with prescribed minimal period for a class of second-order Hamiltonian systems with even potentials. By using the variational methods, we obtain some new existence theorems which improve some recent results in the literature.

Keywords: Minimal period, Critical points, Second-order Hamiltonian systems, Least action principle
2010 MSC: 35A15, 35B10, 35B38, 35J20

1. Introduction and main results

We consider the existence of subharmonic solutions with prescribed minimal period for the following second order Hamiltonian systems

$$\begin{cases} \ddot{u} + V'_x(t, u) = 0, \\ u(0) = u(pT),\ \dot{u}(0) = \dot{u}(pT), \end{cases} \tag{1}$$

where $p \in \mathbb{N}$ with $p > 1$, $V \in C^1(\mathbb{R} \times \mathbb{R}^N, \mathbb{R})$ and $V'_x(t, x)$ denotes the gradient of $V(t, x)$ in x.

As for the existence of subharmonics with minimal periods for systems (1) the pioneer work should trace back to [1]. Applying perturbation type techniques, Birkhoff and Lewis established the existence of a sequence of subharmonics with arbitrarily large minimal period. Using calculus of variations approach, Rabinowitz [2] obtained the existence of nonconstant prescribed periodic solutions of (1) by a global approach. Moreover, Rabinowitz conjectured that system (1) possesses a nonconstant solution with any prescribed minimal period under his conditions. From then on, a vast literature on the minimal period problem for Hamiltonian systems via the critical point theory has amassed, see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and references therein.

Especially, by using the estimate of the energy (the value of the functional associated with the problem) of a solution in terms of the minimal period of the solution, Wang, Wang and Shi [13] obtained explicit sufficient conditions for the existence of subharmonic solutions of (1). These conditions are very easy to check. Specially, $V(t, x)$ is even-typed, that is, $V(-t, -x) = V(t, x)$ for any $(t, x) \in \mathbb{R} \times \mathbb{R}^N$. Moreover, V is subquadratic at infinity, that is,

$$\lim_{|x|\to\infty} \frac{V(t, x)}{|x|^2} = 0 \qquad \text{uniformly in } t. \tag{2}$$

This approach, initially used in [5], has been successfully applied to the minimal period problem of Hamiltonian systems [5, 6, 12, 13, 15].

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Email address: Lch1999@swu.edu.cn (Chun Li)