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A new and sharper bound for Legendre expansion of differentiable functions

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Abstract

In this paper, we present a new and sharper bound for the Legendre coefficients of differentiable functions and then provide a new bound on the approximation error of the truncated Legendre series in the uniform norm. An illustrative example is provided to demonstrate the sharpness of our new results.

Keywords: Legendre coefficient, differentiable functions, sharp bound.

AMS classifications: 41A25, 41A10

1 Introduction

The set of Legendre polynomials $\{P_0(x), P_1(x), \cdots\}$ form a system of polynomials orthogonal on the interval $[-1,1]$ with respect to the weight function $\omega(x) = 1$ and

$$\int_{-1}^{1} P_n(x)P_m(x)dx = h_n\delta_{mn}, \tag{1.1}$$

where δ_{mn} is the Kronecker delta and

$$h_n = \left(n + \frac{1}{2}\right)^{-1}. \tag{1.2}$$

The Legendre expansion of a function $f := [-1,1] \to \mathbb{R}$ is defined by

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x), \quad a_n = h_n^{-1}\int_{-1}^{1} f(x)P_n(x)dx. \tag{1.3}$$

The problem of estimating the magnitude of the Legendre coefficients a_n is of particular interest both from the theoretical and numerical point of view. Indeed, it is useful not only in understanding the rate of convergence of Legendre expansion but useful also in estimating the degree of the Legendre polynomial approximation to $f(x)$ within a given accuracy. When $f(x)$ is analytic in a neighborhood of the interval $[-1,1]$, we note that the estimate of the Legendre coefficients, or more generally, the Gegenbauer and Jacobi coefficients, has been studied in [6, 7, 8, 9].

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