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Nonlinear waves in the modulation instability regime for the fifth-order nonlinear Schrödinger equation

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Abstract

We study the linear stability analysis and nonlinear excitations on a continuous wave background for the fifth-order nonlinear Schrödinger equation. We find that there is a modulation stability (MS) quasi-elliptic ring and a MS curve in the modulation instability (MI) regime. The concrete expressions of different types of nonlinear waves are obtained by using Darboux transformation. And we present the relations and phase diagram between MI and these waves. We discover that the antidark (AD) soliton and nonrational W-shaped soliton can only exist on the resonance line of outside of the MS quasi-elliptic ring and the right of the MS curve. We perform numerical simulation to test the stability of the AD soliton. By introducing the perturbation energy, we further show that solitons in the MI regime are caused by both the fourth- and fifth-order effects.

Keywords: Higher-order effects, Modulation instability, Nonlinear waves, Numerical simulation, Perturbation energy

1. Introduction

The nonlinear Schrödinger equation (NLSE) is a fundamental model for the dynamics and propagation of nonlinear waves, both in optics [1] and in deep water [2]. However, this equation contains only the lowest-order dispersion and lowest-order nonlinearity [3]. If the features of the solution are beyond the simple approximation used to derive the NLSE, higher-order effects will play an important role [4–6]. For example, the third-order effect is vital in the femtosecond regime [7], and the role of the fourth-order effect in an anisotropic Heisenberg ferromagnetic spin has been investigated [8]. In particular, as the intensity of the optical field increases and the duration of the pulse is reduced to the duration of attosecond [9], the quintic effect is becoming increasingly significant. Recent studies have showed that the higher-order effects have influence on the modulation instability (MI) [6] and can induce some novel nonlinear excitations [5, 10].

The MI, namely, the instability of a constant background about the perturbations of long wavelength, is one of the most common types of instabilities in nature [11]. In the context of hydromechanics, it is called as the Benjamin-Feir instability [12]. Additionally, MI was also found in other physical fields, such as nonlinear optics [13], plasma physics [14] and Bose-Einstein condensates [15]. The rogue waves (RWs), which are localized both in space and time [16–19], appear as a result of MI. Recently, some scholars have reported the relations between MI and nonlinear excitations in several integrable systems, such as the NLSE [20], Hirota equation [7], and Lakshmanan-Porsezian-Daniel equation (LPDE) [8]. These studies show that there is a close link between the higher-order effects and MI. For the fifth-order NLSE, Chowdury et al. have presented the exact expressions of several types of nonlinear waves, including the breather and RW solutions [21], as well as soliton solutions [22], (including two-soliton collisions, the degenerate case of the two-soliton solution, and beating structures composed of two or three solitons). Further, they have shown the transformation between breather and W-shaped soliton [23, 24], breather superposition [21] and soliton collision on a constant background [23]. However, little attention is paid to the classification of different kinds of nonlinear wave solutions, and the

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