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Virtual Element Method for Simplified Friction Problem

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Abstract. This work aims at studying the virtual element method (VEM) to solve a simplified friction problem, which is a typical elliptic variational inequality of the second kind. An optimal error estimate is derived in the H^1 norm for the lowest-order VEM. A numerical example is reported to demonstrate the theoretically predicted convergence order.

Keywords. Variational inequality; polygonal meshes; optimal error estimate.

AMS Classification. 65N30, 49J40

1 Introduction

In industry and daily life, friction phenomena between different bodies play an important role in structural and mechanical systems. In this paper, we consider a simplified friction problem, which is a variational inequality of the second kind, featured by the presence of non-differentiable terms in the formulation ([13, 16, 4]). Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with a Lipschitz boundary $\Gamma = \partial\Omega$ that is divided into three parts $\overline{\Gamma}_D$, $\overline{\Gamma}_F$ and $\overline{\Gamma}_C$ with Γ_D, Γ_F and Γ_C relatively open and mutually disjoint such that meas$\,(\Gamma_C) > 0$.

A simplified friction problem. Let $f \in L^2(\Omega)$, $g \in L^2(\Gamma_F)$, and $\eta \in L^2(\Gamma_C)$ with $\eta > 0$. The simplified friction problem is: Find $u \in V := \{v \in H^1(\Omega), v = 0 \text{ on } \Gamma_D\}$ such that

$$a(u, v-u) + j(v) - j(u) \geq \ell(v-u) \qquad \forall\, v \in V, \tag{1.1}$$

where

$$a(u,v) = \int_\Omega (\nabla u \cdot \nabla v + u\, v)\, dx, \qquad \ell(v) = \int_\Omega f\ v\, dx + \int_{\Gamma_F} g\ v\, ds, \qquad j(v) = \int_{\Gamma_C} \eta\, |v|\, ds.$$

This simplified friction problem has a unique solution ([13, 4]). By introducing a Lagrangian multiplier $\lambda \in \Lambda = \{\lambda \in L^\infty(\Gamma_C) : |\lambda| \leq 1 \text{ a.e. on } \Gamma_C\}$, the inequality problem (1.1) can be rewritten as ([4])

$$a(u,v) + \int_{\Gamma_C} \eta\, \lambda\, v\, ds = \ell(v) \qquad \forall\, v \in V, \tag{1.2}$$

$$\lambda\, u = |u| \qquad \text{a.e. on } \Gamma_C. \tag{1.3}$$

The Finite element method (FEM) is a natural numerical discretization approach for variational inequalities [14, 9, 16, 19]. The classical FEM works on the elements with simple geometries, like triangles and rectangles. Due to the flexibility on constructing the local function space, the discontinuous Galerkin (DG) method can handle very general meshes with hanging nodes, which make them very suitable for *hp* adaptivity. However, the DG method needs large number of degrees of freedom ([12, 22, 23, 17, 24, 25]). Recently,

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