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Interval oscillation criteria for nonlinear differential equations with impulses and variable delay

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Abstract: In this paper, the interval qualitative properties of a class of second order nonlinear differential equations are studied. For the hypothesis of delay $\tau(t)$ being variable, the definitions of "strong zero point" and "weak zero point" are introduced and their effects on the estimation of function $x(t-\tau(t))/x(t)$ on each considered interval are investigated, then Riccati transformation and ω functions are applied to obtain interval oscillation criteria. The known results gained by Huang and Feng (2010) for $\tau(t)$ being constant and by Zhou and Wang (2016) for $\tau(t)$ being variable are developed.

Keywords: Interval oscillation, Impulse, Variable delay, Interval delay function.

2010 Mathematics Subject Classification: 34K11, 34A37, 65L03.

1 Introduction

In this paper, we consider the following second order nonlinear impulsive differential equations

$$\begin{aligned} &x''(t)+p(t)g(x(t-\tau(t)))=f(t), \quad t\ \geq t_0, \quad t\neq\theta_k, \\ &x(t^+)=a_kx(t),\ x'(t^+)=b_kx'(t), \quad t=\theta_k, \quad k=1,2,\dots \end{aligned} \tag{1.1}$$

where $\{\theta_k\}$ denotes the impulsive moments sequence with $0\leq t_0<\theta_1<\theta_2<\cdots<\theta_k<\cdots$ and $\lim_{k\to\infty}\theta_k=\infty$.

From the Sturm Separation Theorem, oscillation is only an interval property. Interval oscillation is the embodiment of this idea, i.e., if there exists a sequence of subintervals $[a_i,b_i]$ of $[t_0,\infty)$, as $a_i\to\infty$, such that for each i there is a nontrivial solution of the considered equation which has at least two zeros in $[a_i,b_i]$, then every solution is oscillatory, no matter what the behavior of the coefficients of the equation is on the remaining parts of $[t_0,\infty)$. For this reason, many researchers [1-15] have focused on interval oscillations in the past few decades.

In 2016, by discussing zero points of an "interval delay function" $D_k(t)=t-\theta_k-\tau(t)$ on intervals of impulse moments, Zhou and Wang [5] estimated the function $x(t-\tau(t))/x(t)$ on each considered interval and established some interval oscillation criteria of Eq. (1.1). They investigated the effect of variable delay $\tau(t)$ upon interval oscillation of Eq. (1.1) and developed the work of Huang and Feng [6] with the assumption of delay $\tau(t)$ being constant.

As is known to all, the study of interval oscillation of Eq. (1.1) for $\tau(t)$ being variable is much more difficult than that for $\tau(t)$ being constant. To overcome this difficult, the authors of [5] gave an assumption A: $D_k(t)$ has at most one zero point on (θ_k,θ_{k+1}) for any $k=1,2,...$ (cf. (A_4) in [5]). However, As pointed out in Remark 2.1 in [5], the situation for zero points of $D_k(t)$ on (θ_k,θ_{k+1}) may be very complicated, assumption A is just a simple case of $D_k(t)$. So, how other complex situations influence interval oscillation of Eq. (1.1) is worth investigating.

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