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Finite difference/spectral-Galerkin method for a two-dimensional distributed-order time-space fractional reaction-diffusion equation ☆

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Abstract

In this letter, we consider the numerical approximation of a two-dimensional distributed-order time-space fractional reaction-diffusion equation. The time- and space-fractional derivatives are considered in the senses of Caputo and Riesz, respectively. By using the composite mid-point quadrature, the original fractional problem is approximated by a multi-term time-space fractional differential equation. Then the multi-term Caputo fractional derivatives are discretized by the $L2\text{-}1_\sigma$ formula. We apply the Legendre-Galerkin spectral method for the spatial approximation. Two numerical experiments with smooth and non-smooth initial conditions, respectively, are performed to illustrate the robustness of the proposed method. The results show that: our scheme can arrive at the spectral accuracy (resp. algebraic accuracy) in space for the problem with smooth (resp. non-smooth) initial condition. For both of these two cases, our scheme can lead to the second-order accuracies in time. Additionally, the convergence rates in both spatial and temporal distributed-order variables are two.

Keywords: Distributed-order fractional derivative; Reaction-diffusion equation; Finite difference; Spectral approximation

1. Introduction

Recently, the fractional differential equations (FDEs) have been extensively studied because of their wide applications in science and engineering [1]-[4]. Among kinds of FDEs, the distributed-order ones, firstly introduced by Chechkin *et al.* [5], have been successfully used to describe the anomalous diffusion and relaxation phenomena for which the diffusion exponent can change in the course of time. After this pioneering work, there has been a growing interest in constructing numerical methods for the distributed-order FDEs.

To solve the distributed-order time-fractional differential equations, Mashayekhi and Razzaghi [6] proposed a numerical scheme based on the block-pulse functions and Bernoulli polynomials. Chen *et al.* [7] numerically solved a one-dimensional distributed-order time-fractional reaction-diffusion equation on an unbounded domain. Based on the numerical quadrature formula and reproducing kernel method, Li and Wu [8] presented the numerical solutions of the distributed-order time-fractional diffusion equation with variable coefficients.

For the distributed-order space-fractional differential equations, Liu *et al.*[9] constructed a finite volume method to solve such problem. Then, Fan and Liu [10] applied the finite element method for a two-dimensional distributed-order space-fractional diffusion equation on an irregular convex domain. Jia and Wang [11] developed a fast finite difference method for the distributed-order space-fractional partial differential equation on convex domains.

To the best of our knowledge, no published paper takes into account the numerical solutions of FDEs involving the distributed-order fractional derivatives both in time and in space. This gap in the research literature is the motivation

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