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MULTIPLE SOLUTIONS FOR DEGENERATE NONLOCAL PROBLEMS

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ABSTRACT. In this paper, we study the existence of solutions for a class of degenerate nonlocal problems involving sub-linear nonlinearities, while the nonlinear part of the problem admits some hypotheses on the behavior at the origin or perturbation property. We obtain some new criteria for existence of two and infinitely many solutions of the problem using critical point theory. Some recent results are extended and improved. Some examples are presented to demonstrate the application of our main results.

1. INTRODUCTION

In this paper we consider the existence of multiple nontrivial weak solutions for nonlocal degenerate problems of the form

$$\begin{cases} -M\left(\int_\Omega |x|^{-ap}|\nabla u(x)|^p \mathrm{d}x\right) \operatorname{div}\left(|x|^{-ap}|\nabla u|^{p-2}\nabla u\right) = \lambda |x|^{-p(a+1)+c} f(u), & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $0 \le a < \frac{N-p}{p}$, $1 < p < N$, $c > 0$, $\lambda > 1$, and $M : \mathbb{R}^+ \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are two continuous functions.

In recent years, much attention has been dedicated to the study of nonlocal operators of elliptic type which is modeled by the singularity at infinity. From a physical viewpoint, nonlocal operators play a significant role in describing a set of phenomena. As a general reference here, we can cite the recent paper of Vázquez [25], where the author discusses two models of flow in porous media, including nonlocal (long-range) diffusion effects, and provides a long list of references related to physical phenomena and nonlocal operators. The first model is based on Darcy's law, and the pressure is associated with the density by an inverse fractional Laplacian operator. The second model mostly follows fractional Laplacian flows but it is nonlinear. In contrast to the usual porous medium flows, it has infinite speed of propagation.

The Kirchhoff model takes into consideration the length changes of the string produced by transverse vibrations (see [13]). The solvability of Kirchhoff-type problems has been analyzed by many scholars. Some earlier investigations can be observed in the articles [1, 10, 11, 12, 15, 16, 18, 20, 23] and the related references.

A detailed study is presented in the recent book [22]. Using the minimization technique and the maximum principle, Brézis and Oswald in [3] found an existence and uniqueness result for the p-Laplacian type problem $-\operatorname{div}(a(x, \nabla u)) = f(x, u)$ in Ω and $u = 0$ on $\partial\Omega$. when the behaviour of $f(s)/s$ is properly controlled at infinity. In [6], Chang and Toan used variational methods to analyze this problem . While $f(x, u) = h(x)|u|^{q-2} + g(x)$ and $\Omega = \Omega_1 \times \Omega_2 \subset \mathbb{R}^N$ is a bounded domain having cylindrical symmetry, $\Omega_1 \subset \mathbb{R}^m$ is a bounded regular domain and Ω_2 is a k-dimensional ball of radius R is centered at origin and $m + k = N$, $m \ge 1$, $k \ge 2$. They proved that this problem in this case has at least one or two solutions when $g = 0$ or $g \ne 0$ and some proper conditions on the functions a and h are imposed. It is necessary to have in mind that a special case of the operator $\operatorname{div}(a(x, \nabla u))$ is the p-Laplacian. In [27], Yang et al. proved that (1.1) has at least three distinct weak solutions under some mild assumptions about a. They also proved that it has some of the growth and singularity conditions of f. In [17], Molica Bisci and Rădulescu came up with an existence result for a class of singular quasi-linear elliptic Dirichlet problems on a smooth bounded domain

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