Accepted Manuscript

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PII: S0893-9659(18)30122-8
DOI: https://doi.org/10.1016/j.aml.2018.04.012
Reference: AML 5495

To appear in: *Applied Mathematics Letters*

Received date : 27 January 2018
Revised date : 10 April 2018
Accepted date : 10 April 2018

Please cite this article as: D. Shi, P. Mu, H. Yang, Superconvergence analysis of a two-grid method for semilinear parabolic equations, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.04.012

Superconvergence analysis of a two-grid method for semilinear parabolic equations

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Abstract

In this paper, the superconvergence analysis of a two-grid method (TGM) is established for the semilinear parabolic equations. Based on the combination of the interpolation and Ritz projection technique, an important ingredient in the method, the superclose estimates in the H^1-norm are deduced for the backward Euler fully-discrete TGM scheme. Moreover, through the interpolated postprocessing approach, the corresponding global superconvergence result is derived. Finally, some numerical results are provided to confirm the theoretical analysis, and also show that the computing cost of the proposed TGM is only half of the conventional Galerkin finite element methods (FEMs).

Keywords: Superclose and superconvergence results, TGM, Semilinear parabolic equations, Galerkin FEMs.

1. Introduction

In this paper, we consider the following semilinear parabolic equations:

$$\begin{cases} u_t - \Delta u = f(u), & (X,t) \in \Omega \times J, \\ u = 0, & (X,t) \in \partial\Omega \times J, \\ u(0) = u_0, & (X,t) \in \Omega \times \{t = 0\}, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with Lipschitz boundary $\partial\Omega$, $X = (x, y)$, $J = (0, T]$ and $u_t = \frac{\partial u}{\partial t}$. $f(\cdot)$ is twice continuously differentiable and u_0 is a given function.

There have been some works about the problem (1). For example, the mathematical theory of FEMs for the parabolic PDEs were discussed in [1]. Some optimal error estimates were given in both the H^1-norm and the L^2-norm by a new weak Galerkin method in [2]. Three Crank-Nicolson-type immersed FEMs were developed in [3] for solving the parabolic equations whose diffusion coefficients are discontinuous across a time dependent interface. Unconditional superconvergence analysis for a nonlinear parabolic problem was investigated by a linearized Galerkin FEM in [4], and the mixed FEMs were studied in [5, 6].

On the other hand, the TGM is usually regarded as an efficient discretization technique for solving the nonlinear equations. Its basic idea is that a nonlinear or non-symmetric indefinite problem on the coarse mesh with size H is solved, and then a linearized problem (one Newton like iteration) through the solution obtained from the coarse grid on the fine mesh with size h ($h \ll H$) is dealt with (see [7–9]). Later on, a two-grid finite volume method was studied for the problem (1) in [10, 11]. [12] proposed two kinds of two-grid finite difference methods and established the error estimates in the L^2-norm for the problem (1). Unconditional optimal error estimates of order $O(h + H^3 + \tau)$ in the H^1-norm and order $O(h^2 + H^3 + \tau)$ in the L^2-norm were deduced in [13] by introducing a corresponding time-discrete system. Furthermore, the TGM also has been applied to the semilinear reaction-diffusion equations [14], eigenvalue problems [15], Navier-stokes equations [16–18], Maxwell's equations [19] and so on.

However, the above studies only considered the convergence behavior of the corresponding methods studied. In this paper, as a first attempt, we take the bilinear finite element for example to develop a two-grid algorithm for the problem (1) and investigated the corresponding superclose and superconvergence analysis. By use of the combination of the interpolation and Ritz projection skill employed in [20], the superclose and superconvergent estimates derived herein are under a weaker hypothesis of $u_t \in L^2(J; H^2(\Omega))$ instead of $u_t \in L^2(J; H^3(\Omega))$ required in [21, 22]. It seems that the results obtained in the present work have never been seen in the existing literature.

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