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Infinitely many positive solutions for a nonlocal problem *†

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Abstract: In this paper, we obtain infinitely many small positive solutions of the following nonlocal problem

$$\begin{cases} -\mathcal{L}_K u = f(x,u) \text{ in } \Omega\,, \\ u = 0 \ \text{ in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary $\partial\Omega$, and $\mathcal{L}_K$ is an integrodifferential operator of fractional Laplacian type. The character of this work is that we do not require any growth condition on f for u large.

Keywords: nonlocal problem; fractional Laplacian; positive solution; variational method.

1 Introduction

In the present paper, we are concerned with the following nonlocal equation

$$\begin{cases} -\mathcal{L}_K u = f(x,u) \text{ in } \Omega\,, \\ u = 0 \ \text{ in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary $\partial\Omega$, and $\mathcal{L}_K$ is an integrodifferential operator of fractional Laplacian type and is defined as (see e.g. [15])

$$\mathcal{L}_K u(x) := \int_{\mathbb{R}^N} [u(x+y) + u(x-y) - 2u(x)]K(y)dy, \ x \in \mathbb{R}^N, \tag{1.2}$$

where the kernel $K : \mathbb{R}^N \setminus \{0\} \to (0, +\infty)$ satisfies

(K_1) $\gamma K \in L^1(\mathbb{R}^N)$, where $\gamma(x) = \min\{|x|^2, 1\}$,

(K_2) there exist $\delta > 0$ and $s \in (0,1)$ with $s < N/2$ such that $K(x) \geq \delta|x|^{-(N+2s)}$ for all $x \in \mathbb{R}^N \setminus \{0\}$.

Note if $K(y) = |y|^{-(N+2s)}$, then $\mathcal{L}_K$ is the so-called fractional Laplacian operator $(-\Delta)^s$, which plays an increasingly significant role in the anomalous diffusion ([13] and [1]), the dynamics of the dislocation of atoms in crystals [5], and the fractional quantum mechanics [11]. Different from the operator $-\Delta$, the fractional Laplacian operator $(-\Delta)^s$ or $\mathcal{L}_K$ is nonlocal, and this brings some essential difference with the elliptic equations with the classical Laplacian operator, such as regularity, Maximum principle and so on. This difficulties prevents us from using the variational methods in a standard way. The variational settings of Eq. (1.1) were established in [15, 16, 17, 18, 7]. We refer to [3, 6, 14, 8, 9, 20, 19, 21, 25] and the references therein for more related results. Especially, in [25] and [14], the authors obtained infinitely many solutions for (1.1) with f superlinear at infinity.

On the other hand, when $K(y) = |y|^{-(N+2s)}$ with $s = 1$, Eq. (1.1) turns into a semilinear elliptic equation

$$\begin{cases} -\Delta u = f(x,u) \quad \text{in } \Omega\,, \\ u = 0 \text{ on } \partial\Omega. \end{cases} \tag{1.3}$$

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