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Analysis of a fractional SEIR model with treatment

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Abstract

In this paper we focus on models consisting of fractional differential equations to describe the dynamic of certain epidemics. The population is divided into susceptible, exposed, infectious, and recovered (SEIR), with treatment policies. We present an analytical study and show that the model has two equilibrium points (disease free equilibrium and endemic equilibrium). Local asymptotic stability is proven for both cases. Numerical simulations are presented to illustrate the conclusions.

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1 Introduction

For the reader's convenience we will present a brief overview of fractional calculus, following [17]. Given a positive real number α and an integrable function $x : [a, b] \to \mathbb{R}$, the fractional integral of x of order α is defined as

$$I^\alpha_{a+}x(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} x(\tau)d\tau, \quad t > a.$$

It is worth to mention that when α is an integer number, $I^\alpha_{a+}x$ is simply an n-tuple integral. For what concerns fractional derivatives, although there exist several definitions, we will work with the Caputo fractional derivative, defined as

$${}^C D^\alpha_{a+}x(t) = \left(\frac{d}{dt}\right)^n I^{n-\alpha}_{a+}\left[x(t) - \sum_{k=0}^{n-1} \frac{x^{(k)}(a)}{k!}(t-a)^k\right], \quad t > a,$$

where $n = [\alpha] + 1$. When α is an integer number, ${}^C D^\alpha_{a+}x$ is the usual derivative of order α of x. If x is a function of class C^n, its fractional derivative is represented by the expression

$${}^C D^\alpha_{a+}x(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} x^{(n)}(\tau)d\tau, \quad t > a.$$

In opposite to ordinary differentiation, fractional derivative is a non-local operator and because of it this may be more suitable to study long time behaviour of the function. The operator ${}^C D^\alpha_{a+}$ satisfies the following two basic properties:

$${}^C D^\alpha_{a+} I^\alpha_{a+}x(t) = x(t) \quad \text{and} \quad I^\alpha_{a+}\, {}^C D^\alpha_{a+}x(t) = x(t) - \sum_{k=0}^{n-1} \frac{x^{(k)}(a)}{k!}(t-a)^k, \quad t > a.$$

Fractional differential equations (FDEs) are an extension of ODEs, where the integer order derivative is replaced by a fractional derivative. There has been a significant development in fractional differential equations in recent years due its applicability in different fields of science and engineering. FDEs can help us to reduce the errors arising from the neglected parameters in modeling real life phenomena. Many mathematical modelings contain FDEs. To mention a few, fractional derivatives are used in diffusion equations [19], mechanics [20], viscoelastic properties of many polymeric materials [12], and decision-making models [21]. Also, FDEs may be useful to describe infectious disease dynamics. In [10] is studied a fractional order model for childhood diseases and vaccines, and in [6] the Robbes disease. The HBV infection