Accepted Manuscript

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PII: S0893-9659(18)30129-0
DOI: https://doi.org/10.1016/j.aml.2018.04.019
Reference: AML 5502

To appear in: *Applied Mathematics Letters*

Received date : 14 March 2018
Revised date : 22 April 2018
Accepted date : 22 April 2018

Strong and weak property of travelling waves for degenerate diffusion/aggregation-diffusion models with non-smooth reaction term[1]

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Abstract

In this paper, we investigate a degenerate reaction-diffusion/reaction-aggregation-diffusion model with non-smooth reaction term. The strong and weak property of travelling wave front solutions for this model is analyzed. Under degenerate reaction-diffusion/reaction-aggregation-diffusion term and non-smooth reaction term, we obtain how strong and weak property changes.

Keywords: Reaction-diffusion, reaction-aggregation-diffusion, traveling wave solution, degenerate diffusion.

1. Introduction

By using a random walk approach, Skellam [1] derived the reaction-diffusion equation

$$u_t = (Du_x)_x + g(u), \tag{1}$$

where $u(x,t)$ denotes the population density at position x and time t, $D > 0$ represents the diffusion coefficient and the reaction term $g(u)$ accounts for the rate at which individuals enter the population due to births and deaths (i.e., the net rate of growth like $u(u-1)$). However, it was realized that model (1) failed to consider the different behavioral features of the individuals of the population (see [2, 3]). In general, if the motion is not influenced by neighbors, the motion is said to be density-independent, and it is said to be density-dependent if it is affected by the presence of other individuals. Among these are positive density-dependent diffusion models, i.e., the diffusion coefficient as a positive function of u, which describe the phenomenon that species frequently migrate to regions of lower density more rapidly as the populations are more crowded (see [4, 5, 6]).

In addition, motivated by the need for survival, mating or to overcome a hostile environment, the population have a tendency of aggregation when the density u is small. Due to social behavior or defense against predators, the individual of the species will attract other individuals of the same species (see [7, 8, 9, 10, 11]). Bao and Zhou [12] derived the population model which is aggregating when the population density is small and diffusion when the density is large. In this case, the term $D(u)$ changes its sign once, from negative to positive values, in the interval $u \in [0,1]$, i.e., $D(u) < 0$ for $u \in (0,\alpha)$ and $D(u) > 0$ for $u \in (\alpha, 1)$. In other words, when the density is small, aggregation prevails, while for population densities beyond the critical value α the tendency to dispersal becomes predominant.

Meanwhile, an interesting phenomenon occurs when the density-dependent diffusion coefficient is degenerate, meaning that diffusion approaches zero when the density does also (see [13, 14, 15]). Reaction-diffusion models with degenerate density-dependent diffusion are widely used to describe biological phenomena (see, for example, [16, 17] and the references therein). It is clear that the degenerate diffusion presents an interesting peculiarity: the travelling wave solutions corresponding to the critical speed of propagation is finite.

A series of papers ([16, 18, 19]), replaced the classical logistic growth of equation (1) by a generalized form $g(u) = au^n(1-u)$ where $a > 0$ and $n > 0$. For populations, high decay rates of low density are frequently observed

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