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# Regularized DPSS preconditioners for non-Hermitian saddle point problems

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**Abstract**

Based on a new regularized deteriorated positive and skew-Hermitian splitting (RDPSS) of the coefficient matrix, a RDPSS preconditioner is introduced for non-Hermitian saddle point problems. From implementation aspects, the RDPSS preconditioner has better computing efficiency than the regularized Hermitian and skew-Hermitian preconditioner studied recently (BIT Numer. Math., 57 (2017) 287-311). It is proved that the corresponding RDPSS stationary iteration method is convergent unconditionally. In addition, clustering property of the eigenvalues of the RDPSS preconditioned matrix is carefully studied.

*Keywords:* non-Hermitian saddle point problems, deteriorated positive and skew-Hermitian splitting, regularization, preconditioning, convergence
*2000 MSC:* 65F10

## 1. Introduction

In many scientific computing and engineering applications, we need to solve the following large sparse non-Hermitian saddle point problems

$$\mathcal{A}u \equiv \begin{pmatrix} A & B^* \\ -B & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \equiv b, \tag{1.1}$$

where $A \in \mathbb{C}^{n\times n}$ is non-Hermitian positive definite, $B \in \mathbb{C}^{m\times n}$ $(m \le n)$ is a rectangular matrix of full row rank, $B^*$ denotes the conjugate transpose of $B$, and $f \in \mathbb{C}^n$ and $g \in \mathbb{C}^m$ are two given vectors. These assumptions entail that the non-Hermitian saddle point matrix $\mathcal{A}$ is nonsingular and the solution of (1.1) exists and is unique [1, 2]. For the background information of the linear systems (1.1), please see [2] and references therein.

For large scale linear systems, iterative methods must be used and require then efficient preconditioners. Some good surveys of efficient iterative methods and preconditioning techniques for saddle point problems (1.1) can be found in [2, 3]. Recently, for saddle point problems with Hermitian positive definite (1,1) block matrix, by adding a regularization Hermitian positive semidefinite matrix in the Hermitian and skew-Hermitian splitting (HSS), Bai and Benzi proposed a regularized HSS (RHSS) iteration method and the corresponding RHSS preconditioner in [4]. It is proved that the RHSS iteration method is unconditionally convergent to the unique solution of saddle point problems. In addition, the clustering property of the eigenvalues of the RHSS preconditioned saddle point matrix is described. Thus, it extends the HSS convergence theory [5, 6, 7, 8]. When the RHSS iteration method is applied to solve the non-Hermitian saddle point problems (1.1), the specific iteration scheme is

$$\begin{cases} (\alpha I + \hat{\mathcal{H}}_+)u^{k+\frac{1}{2}} = (\alpha I - \hat{\mathcal{S}}_-)u^k + b, \\ (\alpha I + \hat{\mathcal{S}}_+)u^{k+1} = (\alpha I - \hat{\mathcal{H}}_-)u^{k+\frac{1}{2}} + b, \end{cases} \qquad k = 0, 1, 2, \cdots, \tag{1.2}$$

which is based on the following two matrix splittings

$$\begin{aligned} \mathcal{A} &= \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix} + \begin{pmatrix} S & B^* \\ -B & -Q \end{pmatrix} = \hat{\mathcal{H}}_+ + \hat{\mathcal{S}}_- \\ &= \begin{pmatrix} S & B^* \\ -B & Q \end{pmatrix} + \begin{pmatrix} H & 0 \\ 0 & -Q \end{pmatrix} = \hat{\mathcal{S}}_+ + \hat{\mathcal{H}}_-, \end{aligned}$$

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