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About behaviour of solution of difference equation with a logarithmic nonlinearity under stochastic perturbations

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Abstract

A nonlinear difference equation with delay and logarithmic nonlinearity is considered. Some properties of asymptotic behavior of the solution of this equation under stochastic perturbations are discussed. In particular, stability and instability of the positive and zero equilibria are studied. All obtained results are illustrated by numerical simulations of solutions of the considered equation.

Key words: Difference equation, logarithmic nonlinearity, positive and zero equilibria, stochastic perturbations

1 Introduction and simple difference equation

It is known that nonlinear differential and difference equations play very important role in applications [1–5]. Together with a polynomial and an exponential nonlinearities a logarithmic nonlinearity also is used in a lot of applied mathematical models (see [6] and the references therein). But in difference from a polynomial or an exponential nonlinearity stability analysis of stochastic difference equations with a logarithmic nonlinearity is much more complicated in research. At least no any results in this direction were found by the author. Below, some attempts are made to obtain the first results on the asymptotic behavior of the solution of the difference equation with delay and logarithmic nonlinearity under stochastic perturbations.

Consider for the beginning the simple difference equation with a logarithmic nonlinearity

$$\Delta x_i = \alpha x_i + \mu x_i \ln x_i, \quad i \geq 0, \quad \Delta x_i = x_{i+1} - x_i, \quad x_0 > 0, \tag{1}$$

where α is an arbitrary and μ is a non-zero real parameters, $x_i > 0$, so, $\ln x_i$ is defined.

Via $\lim\limits_{x \to 0} x \ln x = 0$ the equation (1) has both the positive and the zero equilibria

$$x^* = \exp\left(-\frac{\alpha}{\mu}\right), \quad x_0^* = 0. \tag{2}$$

Lemma 1 *If $\mu > 0$ then both equilibria (2) of the equation (1) are unstable. If $\mu < 0$ then the positive equilibrium x^* is locally asymptotically stable and the zero equilibrium is unstable.*

Proof : Let be $\mu > 0$. If $x_i \in \left(\exp\left(-\frac{1+\alpha}{\mu}\right), x^*\right)$, $i = 0, 1, ...$, then via (1), (2) we have

$$x_{i+1} = (1 + \alpha + \mu \ln x_i)x_i < (1 + \alpha + \mu \ln x^*)x_i = x_i, \tag{3}$$