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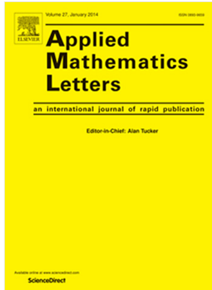

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FRACTIONAL DIFFERENTIAL EQUATIONS INVOLVING GENERALIZED DERIVATIVE WITH STIELTJES AND FRACTIONAL INTEGRAL BOUNDARY CONDITIONS

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ABSTRACT. In this paper, we obtain the sufficient conditions for the uniqueness of solutions for a boundary value problem of fractional differential equations involving generalized fractional derivative supplemented with Stieltjes and generalized fractional integral boundary conditions.

1. INTRODUCTION

Differential equations of fractional order, regarded as generalizations of classical (integer order) differential equations, are gaining immense importance in view of their increasing applications in a variety of fields such as physics, chemical technology, population dynamics, biotechnology, economics, viscoelasticity, control theory of dynamical systems, electrical networks optics and signal processing, rheology etc. For details, we refer the reader to the works [1, 2, 3, 4, 5, 6, 7, 10, 12] and the references cited therein.

Fractional derivatives are defined in terms of fractional integrals and there do exist different definitions of such derivatives depending upon the chosen fractional integral. Among the well known fractional integrals, Riemann-Liouville and the Hadamard fractional integrals occupy central importance in view of their development and applications. A new fractional integral, which unifies the Riemann-Liouville and Hadamard integrals into a single form, was introduced by Katugampola in [8]. This integral is known as *generalized Riemann-Liouville fractional integral* or *Katugampola fractional integral*. The generalized fractional derivative associated with Katugampola fractional integral was introduced in [9] (see definitions in Section 2). Recently, Lupinska and Odzijewicz in [11] obtained a Lyapunov-type inequality for a fractional boundary value problem with Katugampola fractional derivative.

In this paper, we initiate the study of boundary value problems of generalized fractional differential equations supplemented with Stieltjes and generalized fractional integral boundary conditions. In precise terms, we investigate the uniqueness of solutions for the following generalized fractional differential equation:

$$ {}^{\rho}D^{\alpha} y(t) = f(t, y(t)), \ t \in [0, T], \tag{1.1}$$

supplement with Stieltjes and generalized fractional integral boundary conditions of the form

$$ y(0) = 0, \int_0^T y(s)dH(s) = \lambda \frac{\rho^{1-\beta}}{\Gamma(\beta)} \int_0^{\xi} \frac{s^{\rho-1}}{(\xi^{\rho} - s^{\rho})^{1-\beta}} y(s)ds := \lambda \ {}^{\rho}I^{\beta} y(\xi), \ \xi \in (0, T). \tag{1.2}$$

Here ${}^{\rho}D^{\alpha}$ is the generalized fractional derivative of order $1 < \alpha \leq 2, \rho > 0$, ${}^{\rho}I^{\beta}$ is the generalized fractional integral of order $\beta > 0, \rho > 0$, $\int_0^T y(s)dH(s)$ is the Stieltjes integral with respect the function $H : [0, T] \to \mathbb{R}$, $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is a continuous function and $\lambda \in \mathbb{R}$. Here we use the concept of generalized fractional derivative and integral due to Katugampola.

The rest of the paper is organized as follows. In Section 2, we describe the necessary background material related to our problem and prove an auxiliary lemma. Section 3 contains the main result.

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Key words and phrases. Fractional differential equation; generalized fractional integral and derivative; Stieltjes integral; existence; fixed point.