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A remark on Samuelson's variational principle in economics

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ABSTRACT

A general derivation of Euler–Lagrange equation of Samuelson's variational principle in economics is elucidated without Lagrange multipliers, and the optimal solutions and prices can be determined easily.

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1. Introduction

Nature behaves in a way as required by either Hamilton's principle or the least action principle [1–3], and biology behaves in a similar way that metabolism of an animal or a plant follows the following 3/4 scaling law [4,5]

$$B \propto M^{3/4} \tag{1}$$

where B is the organismal basal metabolic rate, M is its organismal mass. The 3/4 scaling law can be derived by maximizing absorbable nutrients through each cell surface [5].

In 1970, Samuelson suggested a variational principle, or a law of conservation of the capital–output ratio for economics similar to that in classical mechanics [6]. The theory was further developed by the owner himself in 1990s [7–9], and it becomes a relatively matured theory in economics thanks to the continuous effort by many authors [10–12].

Samuelson's variational principle [7–11] is to search for an optimal path for a neoclassical von Neumann model with constraints. However, Lagrange multiplier method [13] has difficulty in deriving explicit

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conservation laws. To solve the problem, Kataoka & Hashimoto [10] suggested the Noether theorem for this purpose. In this paper we re-study Samuelson's variational principle [7–11], and reveal that its Euler–Lagrange equation can be obtained without Lagrange multiplier or Noether theorem.

2. Samuelson's variational principle

Samuelson's variational principle [7–11] is to maximize the functional:

$$\max \quad J(K_1, K_2) = \int_0^T \left\{p_1(t)\dot{K}_1(t) + p_2(t)\dot{K}_2(t)\right\} e^{-rt} dt \tag{2}$$

$$\text{s.t.} \quad F(K_1(t), K_2(t), \dot{K}_1(t), \dot{K}_2(t)) = 0 \tag{3}$$

with given initial and terminal conditions:

$$K_i(0) = K_{i0}(i = 1, 2) \tag{4}$$

$$K_i(T) = K_{iT0}(i = 1, 2) \tag{5}$$

where p_1 and p_2 are the prices of goods, $K_i(i = 1, 2)$ is the capital stocks and $\dot{K}_i(i = 1, 2)$ capital formations, r is a constant discount rate, F is a neoclassical smooth transformation function and linear homogeneous with respect to $K_i(i = 1, 2)$ and $\dot{K}_i(i = 1, 2)$,

Economic interpretation is given as follows [7]:

$-\partial F/\partial \dot{K}_i(i = 1, 2)$ and $-\partial F/\partial K_i(i = 1, 2)$ are, respectively, the cost-price and the net rental of the ith capital good, $-\dot{K}_1\partial F/\partial \dot{K}_1 - \dot{K}_2\partial F/\partial \dot{K}_2$ is the national income, and $-K_1\partial F/\partial \dot{K}_1 - K_2\partial F/\partial \dot{K}_2$ is the national wealth.

The variational principle given in Eq. (2) seems to be simple, and its Euler–Lagrange equation can be derived using the Lagrange multiplier method [10,13]:

$$J(K_1, K_2, \lambda) = \int_0^T \left\{e^{-rt}p_1(t)\dot{K}_1(t) + e^{-rt}p_2(t)\dot{K}_2(t) + \lambda F(K_1(t), K_2(t), \dot{K}_1(t), \dot{K}_2(t))\right\} dt \tag{6}$$

where λ is the Lagrange multiplier.

The Euler–Lagrange equations of Eq. (6) are as follows

$$\lambda \frac{\partial F}{\partial K_1} - \frac{d}{dt}\left[e^{-rt}p_1(t) + \lambda \frac{\partial F}{\partial \dot{K}_1}\right] = 0 \tag{7}$$

$$\lambda \frac{\partial F}{\partial K_2} - \frac{d}{dt}\left[e^{-rt}p_2(t) + \lambda \frac{\partial F}{\partial \dot{K}_2}\right] = 0. \tag{8}$$

The Lagrange multiplier is inexplicitly involved in Eqs. (7) and (8), and it is a tedious work to identify the multiplier. To solve the problem, Kataoka & Hashimoto [10] gave a family of transformations $(\tau, \omega, \xi^1, \xi^2)$, given below:

$$\widetilde{t} = t + \varepsilon\tau(t, \lambda, K_1, K_2) \tag{9}$$

$$\widetilde{\lambda} = \lambda + \varepsilon\omega(t, \lambda, K_1, K_2) \tag{10}$$

$$\widetilde{K}_1 = K_1 + \varepsilon\xi^1(t, \lambda, K_1, K_2) \tag{11}$$

$$\widetilde{K}_2 = K_2 + \varepsilon\xi^1(t, \lambda, K_1, K_2) \tag{12}$$

in which ε is an infinite small parameter. Using Noether theorem, Kataoka & Hashimoto [10] obtained the conservation laws for a neoclassical von Neumann model without directly obtaining the Euler–Lagrange equation from Samuelson's variational principle. Sato [11] used Lie groups and related transformations to obtain the invariance-principle and income wealth conservation laws.

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