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Statistical inference for imperfect maintenance models with missing data

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ABSTRACT

The paper considers complex industrial systems with incomplete maintenance history. A corrective maintenance is performed after the occurrence of a failure and its efficiency is assumed to be imperfect. In maintenance analysis, the databases are not necessarily complete. Specifically, the observations are assumed to be window-censored. This situation arises relatively frequently after the purchase of a second-hand unit or in the absence of maintenance record during the burn-in phase. The joint assessment of the wear-out of the system and the maintenance efficiency is investigated under missing data. A review along with extensions of statistical inference procedures from an observation window are proposed in the case of perfect and minimal repair using the renewal and Poisson theories, respectively. Virtual age models are employed to model imperfect repair. In this framework, new estimation procedures are developed. In particular, maximum likelihood estimation methods are derived for the most classical virtual age models. The benefits of the new estimation procedures are highlighted by numerical simulations and an application to a real data set.

1. Introduction

Maintenance management of industrial systems requires to make the best use of the past repair records in order to assess the efficiency of the maintenance actions and the mechanisms of aging. The resulting decision-making process aims to keep the system operational for as long as possible and consists in optimizing the preventive maintenance policy, in improving the maintenance practice and in possibly suggesting a restructuring of the equipments.

A corrective maintenance (CM) is a set of repair tasks which are performed after the appearance of a failure. The quality of a maintenance action is classically assumed to be either perfect or minimal. A perfect repair restores the system as a new one with the same generation technology. A minimal repair brings the system back to operational status, but in the same overall condition as before the failure. Intermediate configurations are referred as imperfect repairs [31]. The present study focuses on a well-spread class of imperfect maintenance models called Virtual Age Models [25]. In particular, the properties of the most common virtual age assumptions are investigated: the Brown–Proschan model [4] and the Arithmetic Reduction of Age models [13].

Estimation procedures, such as the Maximum Likelihood Method, are well established for virtual age models from the complete observation of the maintenance process since the launch of the system. However, it is not uncommon that the data collection is of poor quality: inappropriate reporting equipment, missing or lost information, unrecorded events, etc. This situation arises regularly after the purchase of a second-hand equipment or during a data migration, particularly from paper documents to digital sources. In the following, the maintenance process is assumed to be recorded over an observation window. This configuration is commonly referred as window-censored process [38], window-observation process [40] or interval-censored process [24]. The information at the beginning of the observation can be of multiple forms: the number of failures before the beginning and the calendar time at the beginning can be known or unknown. Inference procedure has been developed for window-observation processes considering minimal repair [7,36,3] and perfect repair [28,38]. Case studies have been derived for a wide range of practical interests such as water urban networks management [27,39], military applications [40], wind turbines [23], medicine [20] and warranty data analysis [35]. All the estimation methods developed above are based on either the renewal theory or the non-homogeneous Poisson theory, corresponding to perfect and minimal repair, respectively. Very few developments have been proposed in the case of imperfect repair. Gasmi et al. [19] investigate the

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situation where the age of the system is set to a known value different from zero at the beginning of the observation. Two natural but reductive simplifications are also employed when using imperfect maintenance models from an observation window: either the system is assumed to be new at the beginning [26] or no repair is assumed to have been performed prior to the beginning [14].

In this study, a review of the estimation procedures using the Maximum Likelihood Method with complete observations is firstly proposed. Each maintenance efficiency is considered: minimal repair, perfect repair and imperfect repair based on virtual age assumptions. Secondly, an overview of the estimation methods from an observation window with minimal and perfect repair is gathered for the first time and is presented along with multiple new extensions. Different situations are considered depending on the knowledge on the system at the beginning of the observation. Thirdly, new developments in the case of window-censored observations and virtual age assumptions are outlined. As the computation of the likelihood function strongly depends on the imperfect repair model, the estimation procedures are detailed for the most common virtual age models, such as the Brown–Proschan model [4] and two Kijima models [25]. In the presence of a small number of window-censored recurrence data, the gain of the new estimation methods is important and the quality of the prognostics, such as the estimation of the Remaining Useful Life, can be enhanced.

The remaining of the paper is organized as follows. In Section 2, the maximum likelihood method is derived for the classical imperfect maintenance models. The context of window-censored process is introduced. In Sections 3 and 4, a review along with new generalizations of the maximum likelihood methods is presented considering minimal and perfect maintenance, respectively. New developments of estimation procedures for virtual age models from an observation window are proposed in Section 5. The specificities of the usual virtual age assumptions are investigated. The benefits of an appropriate modeling compared to a simplified one are highlighted in Section 6 based on simulation studies. An application to a real data set is provided in Section 7 and emphasizes the risk of analyzing estimation results from an inaccurate modeling.

2. General modelling

2.1. Inference with complete information

We consider a simple case of maintenance databases, where failure and maintenance dates are modeled by a counting process and there is only corrective maintenance (repair upon failure) in the past events. The downtime and repair time are assumed to be negligible, hence the resulting counting process is referred as a failure process. The complete counting process $\{N_t\}_{t\geq 0}$ is such that N_t is the number of failures of the system in the time interval $[0, t]$. The complete observations consist of failure times $\{T_i\}_{i\geq 1}$ and inter-failure times $\{X_i\}_{i\geq 1}$. The failure process is equivalently characterized by the random processes $\{N_t\}_{t\geq 0}$, $\{T_i\}_{i\geq 1}$ or $\{X_i\}_{i\geq 1}$ and their distributions are given by the failure intensity defined as:

$$\forall\, t \geq 0, \quad \lambda_t = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P(N_{t+\Delta t} - N_{t^-} = 1 | \mathcal{H}_{t^-}) \tag{1}$$

where $\mathcal{H}_{t^-}$ and N_{t^-} are the past of the failure process and number of failures just before t, respectively. The history $\mathcal{H}_{t^-}$ corresponds to the set of all events which have occurred before t. Before the first failure, the failure intensity is assumed to be a deterministic and continuous function of time $\lambda(t)$, called the initial intensity, which is the failure rate of the first failure time T_1. As industrial systems are supposed to wear out continuously, the initial intensity is traditionally increasing. It is worthwhile to introduce the cumulative hazard rate function Λ, defined as $\Lambda(t) = \int_0^t \lambda(u)du$. The pdf, cdf and reliability function associated with the first time to failure for a new system are, respectively, f, F and R.

The assumptions on the maintenance efficiency need to be presented. Three widespread models are considered:

A. Maintenances are minimal. This is known as the As Bad As Old (ABAO) model, where a repair leaves the system in the same state as before the failure. The failure process is a Non-Homogeneous Poisson Process (NHPP) and its failure intensity is a function of time: $\lambda_t = \lambda(t)$. The most usual NHPP is the Power Law Process (PLP) [32] and the initial intensity is defined as a power of time, similar to a Weibull distribution:

$$\lambda(t) = \alpha\beta t^{\beta-1} \quad \alpha > 0, \quad \beta > 0 \tag{2}$$

The likelihood function associated to the observation of the system during a period $[0, t]$, where n repairs are performed with failure times $(\tau_1, \ldots, \tau_n)$, and considering an initial intensity $\lambda(t)$ is:

$$\mathcal{L}_t^{NHPP}(\tau_1, \ldots, \tau_n) = \prod_{i=1}^{n} \lambda(\tau_i) \times e^{-\Lambda(t)} \tag{3}$$

B. Maintenances are perfect. This is known as the As Good As New (AGAN) model, where a repair restores the system as new. The resulting failure process is a Renewal Process (RP) where the failure intensity follows $\lambda_t = \lambda(t - T_{N_{t^-}})$. The likelihood function associated with the observation of the system during a period $[0, t]$, where n repairs are performed with failure times $(\tau_1, \ldots, \tau_n)$, where f denotes the pdf of the inter-failure times and considering an initial intensity $\lambda(t)$:

$$\mathcal{L}_t^{RP}(\tau_1, \ldots, \tau_n) = \prod_{i=1}^{n} f(\tau_i - \tau_{i-1}) \times e^{-\Lambda(t-\tau_n)} \tag{4}$$

C. Maintenances are imperfect. Imperfect maintenance models [31] reflect intermediate situations between minimal and perfect repair. This study focuses on the most widespread imperfect maintenance models called *virtual age models* defined by Kijima [25]. To define a virtual age model, one needs to define first of all the effective ages, denoted by a sequence of positive random variables $\{A_i\}_{i\geq 0}$ with $A_0 = 0$. Maintenance actions are assumed to have impact on the system's age, resulting in the effective age A_i just after the ith maintenance. Moreover, it assumes that after the ith maintenance, the system behaves as a new one having survived until A_i without being maintained. The conditional distributions of inter-failure times are given by:

$$\forall\, i \geq 1, \quad \forall\, t \geq 0, \quad P(X_{i+1} > t | A_i, X_1, \ldots, X_i) = P(Y > A_i + t \,|\, Y > A_i) \tag{5}$$

where Y is a random variable with the same distribution as the first failure time X_1. The virtual age of the system at time t denoted V_t is the age of the system at the last maintenance plus the time elapsed since $V_t = A_{N_{t^-}} + t - T_{N_{t^-}}$. A virtual age model is therefore completely defined by a particular sequence of effective ages $\{A_i\}_{i\geq 0}$ and by an initial intensity. The resulting failure intensity [13] is $\lambda_t = \lambda(t - T_{N_{t^-}} + A_{N_{t^-}})$.

Large classes of virtual age models have been presented in [16,22,25]. The ABAO and the AGAN cases correspond to $A_i = T_i$ and $A_i = 0$, respectively. More general models have been proposed such as:

- The Arithmetic Reduction of Age model with memory 1 (ARA_1, [13]) is a particular case of Kijima's Type I model [25]. The effect