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Global boundedness in an attraction-repulsion chemotaxis system with logistic source

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Abstract

We study the attraction-repulsion chemotaxis system of parabolic-elliptic type with logistic source: $u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u)$, $0 = \Delta v - \beta v + \alpha u$, $0 = \Delta w - \delta w + \gamma u$, subject to the non-flux boundary conditions in a bounded domain $\Omega \subset \mathbb{R}^n (n \ge 1)$ with smooth boundary, $f(s) \le a - bs^\eta$ for all $s \ge 0$, where constants $\chi, \xi, \eta, \alpha, \delta, \gamma, b > 0$, $a \ge 0$. The global boundedness of solutions to this problem was established in [X. Li and Z.Y. Xiang, On an attraction-repulsion chemotaxis system with a logistic source, IMA J. Appl. Math. 81 (2016) 165–198] for the repulsion domination case $\chi\alpha < \xi\gamma$ with $\eta \ge 1$, the attraction domination case $\chi\alpha > \xi\gamma$ with $\eta > 2$ (or $\eta = 2$, b properly large), and the balance case $\chi\alpha = \xi\gamma$ with $\eta > \frac{1}{2}(\sqrt{n^2+4n} - n + 2)$, respectively. In the present paper we prove for the balance case $\chi\alpha = \xi\gamma$ that the weakened restriction $\eta > \frac{2n+2}{n+2}$ is sufficient to ensure the global boundedness of solutions.

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Keywords: Attraction-repulsion; Boundedness; Logistic source; Chemotaxis

1 Introduction

We consider the following attraction-repulsion chemotaxis system with logistic source

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u), & (x,t)\in\Omega\times(0,T), \\ \tau v_t = \Delta v - \beta v + \alpha u, & (x,t)\in\Omega\times(0,T), \\ \tau w_t = \Delta w - \delta w + \gamma u, & (x,t)\in\Omega\times(0,T), \\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = \dfrac{\partial w}{\partial\nu} = 0, & (x,t)\in\partial\Omega\times(0,T), \\ u(x,0) = u_0(x), \tau v(x,0) = \tau v_0(x), \tau w(x,0) = \tau w_0(x), & x\in\Omega. \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n (n \ge 1)$ is a bounded domain with smooth boundary and $\partial/\partial\nu$ denotes the derivative with respect to the outer normal of $\partial\Omega$, $\tau \in \{0, 1\}$, $\chi, \xi, \beta, \alpha, \delta, \gamma > 0$, $u(x,t)$, $v(x,t)$ and $w(x,t)$ denote the cell density, the chemoattractant concentration, and the chemorepellent concentration, respectively. The nonnegative initial data $u_0 \in C^0(\bar{\Omega})$. The logistic source $f : \mathbb{R} \to \mathbb{R}$ smooth with $f(0) \ge 0$ satisfies

$$f(s) \le a - bs^\eta \quad \text{for all } s \ge 0, \tag{1.2}$$

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