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Existence of multiple solutions to second-order discrete Neumann boundary value problems

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Abstract

By using the invariant set of descending flow and variational method, we establish the existence of multiple solutions to a class of second-order discrete Neumann boundary value problems. The solutions include sign-changing solutions, positive solutions, and negative solutions. An example is given to illustrate our results.

Keywords: Difference equation, Neumann boundary value problem, Sign-changing solution, Invariant set of descending flow

2017 MR Subject Classification: 39A12, 39A23

1 Introduction

In this work, we are concerned with a class of second-order nonlinear difference equations with Neumann boundary conditions described below

$$\begin{cases} -\Delta[p(n-1)\Delta u(n-1)] + q(n)u(n) = kf(n, u(n)), & n \in [1, N], \\ \Delta u(0) = \Delta u(N) = 0. \end{cases} \tag{1.1}$$

Here $k > 0$ is a parameter, Δ is the forward difference operator defined by $\Delta u(n) = u(n+1) - u(n)$. $N > 1$ is a positive integer and $[1, N] = \{1, 2, \cdots, N-1, N\}$. $f : [1, N] \times R \to R$ is continuous in the second variable, $p : [0, N] \to (0, +\infty)$ satisfies $p(0) = p(1)$ and $q : [1, N] \to [0, +\infty)$.

Discrete nonlinear equations play an important role in describing many physical problems, such as nonlinear elasticity theory or mechanics and engineering topics [1]. Equation (1.1) with $k = 1$ has been extensively studied in the literature. For instance, the existence of nontrivial positive solutions was established in [2, 3, 4]. For results on nonlinear difference equations with other types of boundary conditions, we refer the reader to [5, 6] and references therein. When it comes to sign-changing solutions, much of research has been focused on

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