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Lagrangian Actions for Elliptical Solutions of 2-Body and 3-Body Problems with Fixed Energies*

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Abstract

Based on the works of Gordon ([5]) and Zhang-Zhou([9]) on the variational minimizing properties for Keplerian orbits and Lagrangian solutions of Newtonian 2-body and 3-body problems, we use the constrained variational principle of Ambrosetti-Coti Zelati ([1]) to compute the Lagrangian actions on Keplerian and Lagrangian elliptical solutions with fixed energies. We also find an interesting relation between the period and the energy for Lagrangian elliptical solutions with Newtonian potentials.
Key Words: 2 and 3-body problems, Keplerian orbits, Lagrangian solutions, Fixed energy, Lagrangian actions.

MSC(2010) 70G75, 70F07, 70F10.

1 Introduction and Main Results

In [5], Gordon proved that the Keplerian orbits minimize the Lagrangian action of the Keplerian 2-body problems with a fixed period. In [9], Zhang-Zhou generalized the result of Gordon to Newtonian 3-body problem and proved that the Lagrangian elliptical orbits with equilateral configurations minimize the Lagrangian action with a fixed period. In this note, we try to generalize the above problems to the case with a fixed energy.

Consider Keplerian two-body problem with a fixed energy $h < 0$,

$$\begin{cases} \ddot{x}(t) + \nabla V(x) = 0, \\ \frac{1}{2}|\dot{x}|^2 + V(x) = h, \end{cases} \tag{1}$$

where $x \in R^2$,

$$V(x) = \frac{-a}{|x|}, \qquad a > 0. \tag{2}$$

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