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Competitive exclusion for a two-species chemotaxis system with two chemicals

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ABSTRACT

In this paper we consider the following competitive two-species chemotaxis system with two chemicals

$$\begin{cases} u_t = \Delta u - \chi_1 \nabla \cdot (u \nabla v) + \mu_1 u(1 - u - a_1 w), & x \in \Omega,\ t > 0, \\ 0 = \Delta v - v + w, & x \in \Omega,\ t > 0, \\ w_t = \Delta w - \chi_2 \nabla \cdot (w \nabla z) + \mu_2 w(1 - a_2 u - w), & x \in \Omega,\ t > 0, \\ 0 = \Delta z - z + u, & x \in \Omega,\ t > 0 \end{cases}$$

in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ with $n \geq 1$, where $\chi_i \geq 0$, $a_i \geq 0$ and $\mu_i > 0$ $(i = 1, 2)$. For the case $a_1 > 1 > a_2 \geq 0$, it will be proved that if $\chi_1 \chi_2 < \mu_1 \mu_2$, $\chi_1 \leq a_1 \mu_1$ and $\chi_2 < \mu_2$, then the initial–boundary value problem with homogeneous Neumann boundary condition admits a unique global bounded solution and $(u, v, w, z) \to (0, 1, 1, 0)$ uniformly on $\bar{\Omega}$ as $t \to \infty$.

1. Introduction

This paper is concerned with the parabolic–elliptic version of a two-species chemotaxis system with two signals accounting for Lotka–Volterra type competitive interaction

$$\begin{cases} u_t = \Delta u - \chi_1 \nabla \cdot (u \nabla v) + \mu_1 u(1 - u - a_1 w), & x \in \Omega,\ t > 0, \\ 0 = \Delta v - v + w, & x \in \Omega,\ t > 0, \\ w_t = \Delta w - \chi_2 \nabla \cdot (w \nabla z) + \mu_2 w(1 - a_2 u - w), & x \in \Omega,\ t > 0, \\ 0 = \Delta z - z + u, & x \in \Omega,\ t > 0 \end{cases} \tag{1.1}$$

under the homogeneous Neumann boundary conditions

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = \frac{\partial z}{\partial \nu} = 0, \quad x \in \partial\Omega,\ t > 0 \tag{1.2}$$

and the initial conditions

$$u(x, 0) = u_0(x), \quad w(x, 0) = w_0(x), \quad x \in \Omega \tag{1.3}$$

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in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ with $n \geq 1$. Such system is used to describe the movement of two competitive biological species in the influence of the chemical substance produced by the other. In this setting, $u(x,t)$ and $w(x,t)$ denote the densities of two species, respectively, $v(x,t)$ is the concentration of the chemical produced by $w(x,t)$ and the other chemical density secreted by $u(x,t)$ is denoted by $z(x,t)$. The competitive coefficients a_i and the chemotactic sensitivities χ_i $(i=1,2)$ are all nonnegative. The parameters $\mu_1 > 0$ and $\mu_2 > 0$ represent logistic growth rates of species u and w, respectively.

Systems (1.1) and related form are of significant relevance in the understanding of competitive spatio-temporal dynamics in tactically active populations and accordingly have received considerable interest in the recent literature [1–6]. Concerning the initial–boundary value problem (1.1)–(1.3), it is proved in [6] that if $\chi_1\chi_2 < \mu_1\mu_2$, the solution is global in time and bounded for suitably initial data. Moreover, for the weakly competitive case $0 \leq a_1 < 1$ and $0 \leq a_2 < 1$, if $\chi_1 < a_1\mu_1$ and $\chi_2 < a_2\mu_2$, the solution converges to the coexistence equilibrium point $(\frac{1-a_1}{1-a_1a_2}, \frac{1-a_2}{1-a_1a_2}, \frac{1-a_2}{1-a_1a_2}, \frac{1-a_1}{1-a_1a_2})$ in the large time. It is a natural question to ask whether competitive exclusion occurs in the problem (1.1)–(1.3) for partially strong competitive case in the sense that $a_1 > 1 > a_2 \geq 0$.

The main purpose of this paper is to obtain the asymptotic behavior of the classical solution to the problem (1.1)–(1.3) for the case $a_1 > 1 > a_2 \geq 0$. The result is stated as follows.

Theorem 1.1. *Let $a_1 > 1 > a_2 \geq 0$, $\chi_i \in [0,\infty)$ and $\mu_i \in (0,\infty)$ $(i=1,2)$ and satisfy*

$$\chi_1\chi_2 < \mu_1\mu_2, \quad \chi_1 \leq a_1\mu_1, \quad \chi_2 < \mu_2. \tag{1.4}$$

Then for any nonnegative initial data $u_0 \in C^0(\bar{\Omega})$ and $w_0 \in C^0(\bar{\Omega})$ fulfilling $w_0 \not\equiv 0$, the problem (1.1)–(1.3) admits a unique nonnegative global classical solution (u,v,w,z) such that

$$(u(\cdot,t), v(\cdot,t), w(\cdot,t), z(\cdot,t)) \to (0,1,1,0) \quad \text{as } t \to \infty$$

uniformly with respect to $x \in \Omega$.

Remark 1.1. In the case $a_1 > 1 > a_2 \geq 0$, under the condition (1.4), the second population w eventually outcompetes the first one u in the sense that $u \to 0$ and $w \to 1$ as $t \to \infty$.

Remark 1.2. In the special case $\chi_1 = \chi_2 = \chi$ and $\mu_1 = \mu_2 = \mu$, condition (1.4) reduces to $\chi < \mu$.

2. Proof of Theorem 1.1

We first list a statement concerning global boundedness of classical solution for the problem (1.1)–(1.3), which is derived in [6, Theorem 1.1].

Lemma 2.1. *Assume that $a_i, \chi_i \geq 0$, $\mu_i > 0$ for $i = 1,2$ and satisfy $\chi_1\chi_2 < \mu_1\mu_2$. Then for all nonnegative initial data u_0, $w_0 \in C^0(\bar{\Omega})$ fulfilling $w_0 \not\equiv 0$, the problem (1.1)– (1.3) possesses a unique nonnegative global classical solution (u,v,w,z) which is bounded in $\Omega \times (0,\infty)$. Moreover, $v > 0$, $w > 0$ in $\bar{\Omega} \times (0,\infty)$, and either $u \equiv 0$, $z \equiv 0$ or $u > 0$, $z > 0$ in $\bar{\Omega} \times (0,\infty)$.*

Based on the above result, we can define the finite real numbers by

$$L_1 = \limsup_{t\to\infty} \max_{x\in\bar{\Omega}} u(x,t),$$
$$L_2 = \limsup_{t\to\infty} \max_{x\in\bar{\Omega}} w(x,t),$$
$$l_2 = \liminf_{t\to\infty} \min_{x\in\bar{\Omega}} w(x,t).$$