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ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF CERTAIN FORCED THIRD ORDER INTEGRO-DIFFERENTIAL EQUATIONS WITH δ-LAPLACIAN

SAID R. GRACE AND JOHN R. GRAEF

ABSTRACT. In this paper the authors examine the asymptotic behavior of solutions of a certain third order forced integro-differential equations with δ-Laplacian. Their main goal is to investigate whether nonoscillatory solutions behave at infinity like certain nontrivial nonlinear functions. They apply a technique involving Young's, Hölder's, and Gronwall's inequalities.

1. Introduction

Consider the third order forced integro-differential equation

$$\left(a(t)\left|x'(t)\right|^{\delta-1}x'(t)\right)'' + \int_c^t (t-s)^{\alpha-1}k(t,s)f(s,x(s))ds = e(t), \quad c \geq 1, \tag{1.1}$$

where:

(i) $a : [c,\infty) \to (0,\infty)$ and $e : [c,\infty) \to \mathbb{R}$ are continuous functions;

(ii) $k : [c,\infty) \times [c,\infty) \to \mathbb{R}$ is continuous and that there is a positive continuous function $b(t)$ such that
$$|k(t,s)| \leq b(t) \quad \text{for all } t \geq s \geq c;$$

(iii) $f : [c,\infty)\times\mathbb{R} \to \mathbb{R}$ is continuous and there is a continuous function $h : [c,\infty) \to (0,\infty)$ and positive real number γ such that
$$|f(t,x)| \leq h(t)\,|x|^{\gamma} \quad \text{for } x \neq 0 \text{ and } t \geq c;$$

(iv) $\delta \geq 1$ is the ratio of odd positive integers with $\delta > \gamma$, and $0 < \alpha < 1$;

(v) $R(t) = \int_c^t a^{-1/\delta}(s)ds \to \infty$ as $t \to \infty$.

We only consider those solutions of equation (1.1) that are continuable and nontrivial in any neighborhood of ∞. Such a solution is said to be *oscillatory* if it has arbitrarily large zeros, and is *nonoscillatory* otherwise.

In the last few decades, third order differential equations have gained considerably more attention due to their applications in many engineering and scientific disciplines such as mathematical models for systems and processes in fields such as physics, mechanics, chemistry, aerodynamics, and the electrodynamics of complex media. For more details, see for example, [2, 12, 17, 18, 19, 20, 21, 22].

In the asymptotic theory of nonlinear ordinary differential equations, a classic problem is to establish conditions for the existence of solutions approaching polynomials of a certain degree at $t \to \infty$ (see [4, 12, 14, 15, 16, 17, 21, 22]). In the proofs of these kinds of results, a key role is often played by the well-known Bihari inequality.

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