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Planning of step-stress accelerated degradation test based on the inverse Gaussian process

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ABSTRACT

The step-stress accelerated degradation test (SSADT) is a useful tool for assessing the lifetime distribution of highly reliable or expensive product. Some efficient SSADT plans have been proposed when the underlying degradation follows the Wiener process or Gamma process. However, how to design an efficient SSADT plan for the inverse Gaussian (IG) process is still a problem to be solved. The aim of this paper is to provide an optimal SSADT plan for the IG degradation process. A cumulative exposure model for the SSADT is adopted, in which the product degradation path depends only on the current stress level and the degradation accumulated, and has nothing to do with the way of accumulation. Under the constraint of the total experimental budget, some design variables are optimized by minimizing the asymptotic variance of the estimated p-quantile of the lifetime distribution of the product. Finally, we use the proposed method to deal with the optimal SSADT design for a type of electrical connector based on a set of stress relaxation data. The sensitivity and stability of the SSADT plan are studied, and we find that the optimal test plan is quite robust for a moderate departure from the values of the parameters.

1. Introduction

With the rapid development of modern technology and global competition, many new products are designed and manufactured to be more reliable. This makes it difficult for engineers to obtain sufficient failure data within a reasonable life-testing time, and to effectively assess the product's reliability characteristics, such as the reliability function and the mean time to failure (MTTF). The accelerated life test (ALT) techniques are extensively used to acquire sufficient failure data for reliability analysis in the area. However, for some highly reliable products, even subject to high level stress, they are not likely to fail in a short period of time. In this case, the traditional ALT methods are not enough for the reliability evaluation of the products, and the degradation data analysis provides another way for this task. If there exists some quality characteristics of a highly reliable product, which can indicate the healthy state of the product and can be observed during the product degradation process, collecting these degradation data is useful for our statistical inference. Usually, the degradation of a product accumulates over time, and the system fails when the magnitude of the degradation exceeds a given threshold. So, the reliability of the product can be inferred from these degradation data. Compared with the analysis of the lifetime data, the analysis of the degradation data can get the same estimation accuracy with less test samples and test costs [1]. Especially in applications in which few or no failures are expected, degradation data can provide considerably more reliability information than traditional censored failure-time data [2].

For many extremely highly reliable products, such as Hi-tech weaponry, nuclear power plants, and aerospace, the degradation tests conducted under normal operating condition still require a lot of time and cost to acquire enough degradation data for use. Similar to conducting ALT for more failure data, we can use accelerated degradation tests (ADTs) to quickly obtain more degradation data and make some inference for the lifetime of the products. ADTs are able to greatly shorten the testing duration and reduce the test cost, so the research of ADT has attracted much attention in the past decade [3–7]. There are two main classes of models for ADT data analysis. The first one is the general path model [2,8], in which a degradation regression model is assumed and estimated. The other class of models are called stochastic process models, which are used to capture the time-dependent structure of the degradation over time. Two popular stochastic process models are the Gamma process and the Wiener process [5,9–12].

Constant-stress accelerated degradation test (CSADT) is the most popular ADT in application. In a CSADT, the products are divided into several groups, and each group of products are exposed to a certain severe stress condition to collect degradation data. The CSADT usually needs more products to proceed the

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experiment. But for a newly developed product, or an expensive product, it may not be possible to have so many test units on hand. In this situation, the step-stress accelerated degradation test (SSADT) can be used to collect degradation data. For a SSADT, only one group of products is needed to conduct the experiment, and with the prolonging of the time, the products may experience several severe stress conditions successively until the experiment is completed. About statistical inference for data from SSADT or the optimal design of a SSADT experiment, some research work has been done by many scholars recently. Tseng and Wen [13] applied SSADT to assess the reliability of a light emitting diode by using the empirical regression method. Tang et al. [14] designed an optimal SSADT by minimizing the total expected test cost, which is a function of sample size, test duration, and the number of inspections. Liao and Tseng [15] and Tseng et al. [16] provided the optimal SSADT plans by minimizing the variance of estimated p-percentile and the approximate variance of the estimated MTTF, respectively, under constraints on the total cost. More SSADT models can be seen in [17–22].

As we know, the Wiener process and the Gamma process have received widespread applications in degradation data analysis. However, the two models cannot handle all degradation data in real world problems. The IG process is another useful model in dealing with degradation data analysis. For example, Wang and Xu [23] found that neither the Wiener process nor the Gamma process fits the GaAs laser degradation data well, while the IG process model has good performance in fitting these data. EM algorithm is applied to estimate the parameters of the model. A corrosion growth model for energy pipelines was built by using the IG process in Zhang et al. [24], and the MCMC method is used to evaluate the system parameters. Ye and Chen [25] systematically investigated the IG process, and showed that the IG process has many superb properties when dealing with random effects and covariates. In Peng et al. [26], a general Bayesian framework is proposed for degradation analysis based on the IG process, and the impact of the random effects are intensively investigated. Bayesian step stress methods are effective and significant methods because they contain subjective information and historical information which can be incorporated to complement the insufficiency of sparse or fragmented observations. About the related works using Bayesian step stress methods, the readers can refer to [27–30]. The Kullback–Leibler divergence is a measure of the difference between two probability distributions. Specifically, it is a measure of the information gained from the prior probability distribution to the posterior probability distribution. Bregman divergence is similar to a metric, but it neither satisfies the triangle inequality nor is symmetry. In the step stress accelerated degradation test, the Kullback–Leibler (KL) divergence and Bregman divergence can be used as the objective function when optimizing the design variables. Some reference materials can be found in [27,31–33]. The IG process model is gradually becoming an important approach for degradation data analysis. More works along this direction can refer to [34–36]. Ye et al. [34] considered an accelerated degradation test planning for inverse Gaussian process. They discussed the accelerated degradation test planning for both the inverse Gaussian process without and with random effects. The optimal settings including the test stress levels and the number of samples allocated to each stress level are obtained by minimizing the asymptotic variance of the estimate of a lower quantile. Peng et al. [35] discussed an optimization problem for degradation tests based on the inverse Gaussian process from the perspective of Bayesian statistics. The design variables including sample size and the number of measurements are optimized by minimizing the average pre-poster variance of reliability. Peng [36] proposed a degradation model based on an Inverse Normal-Gamma mixture of an IG process. The MLE's of the model parameters were obtained by using EM algorithm and the estimation of the mean-time-to-failure (MTTF) of product was calculated. Moreover, a lot of real world applications are provided in their study, including the GaAs Laser data, Device-B data from Meeker and Escobar.

To the best of our knowledge, no research focuses on the design of the SSADT for the IG process. Therefore, in this paper, we try to present an optimal SSADT plan for the degradation data based on the IG process. A CE model is assumed for the SSADT, that is, the follow-up degradation path of a product at any time t depends only on the degradation already accumulated and the current stress level, and has nothing to do with the way of cumulation. About the CE models, the readers can refer to [37,38]. In the literature, to design the optimal SSADT experiments, the quantities to be minimized can be the asymptotic variance of the estimated p-percentile of the lifetime distribution, the asymptotic variance of the reliability function, and the asymptotic variance of the MTTF, the total test cost and so on. In this work, we use the asymptotic variance of the estimated p-percentile of the life distribution of the tested product as the objective function. The design variables, including the sample size, the measurement frequency, and the number of measurements, are to be determined by minimizing the objective function under the total cost constraints. The optimization problem is solved for the design variables, and then the optimal SSADT plan is constructed. Finally, the optimal SSADT design for a type of electrical connector based on a set of stress relaxation data is studied by using the proposed method. Some sensitivity analyses of the SSADT plan are also provided, and from these study, we find that the optimal test plan is quite robust for the parameters.

The rest of this paper is organized as follows. In Section 2, we describe the IG process and the SSADT, and make some assumptions on the model. In Section 3, the statistical analysis of the degradation is presented, the optimization problem for the SSADT is constructed and the algorithm for seeking for the optimal solution is also provided. An example is given in Section 4, some statistical results are obtained, and the optimal SSADT plans under different cost constraints and some sensitivity analysis results are provided to show the effectiveness of the proposed method. Finally, some conclusions are made in Section 5.

2. Problem description, and formulation

2.1. The IG process

A stochastic process $\{Z(t), t > 0\}$ is called an IG process if it satisfies:

(i) $Z(0) = 0$ with probability one;
(ii) $Z(t)$ has independent increments, that is, $Z(t_2) - Z(t_1)$ and $Z(s_2) - Z(s_1)$ are independent for all $t_2 > t_1 \geq s_2 > s_1 \geq 0$;
(iii) $Z(t) - Z(s)$ follows the IG distribution $IG(\mu(\Lambda(t) - \Lambda(s)), \eta(\Lambda(t) - \Lambda(s))^2)$, for all $t > s \geq 0$, where $\Lambda(t)$ is a monotone increasing function with $\Lambda(0) = 0$, $\mu > 0$ and $\eta > 0$ are constants, and $IG(a, b)$, $a > 0$, $b > 0$ denotes the IG distribution with probability density function (PDF)

$$f(y; a, b) = \left(\frac{b}{2\pi y^3}\right)^{1/2} \exp\left(-\frac{b(y-a)^2}{2a^2 y}\right), \quad y > 0, \tag{1}$$

and cumulative distribution function (CDF)

$$F(y; a, b) = \Phi\left[\sqrt{\frac{b}{y}}\left(\frac{y}{a} - 1\right)\right] + e^{\frac{2b}{a}}\Phi\left[-\sqrt{\frac{b}{y}}\left(\frac{y}{a} + 1\right)\right], \quad y > 0, \tag{2}$$