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Asymptotic behavior of a weakly dissipative modified two-component Dullin-Gottwald-Holm system☆

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Abstract

In this paper, we consider the asymptotic behavior of a weakly dissipative modified two-component Dullin-Gottwald-Holm (mDGH2) system. We derive the asymptotic behavior of the solution at infinity expressed in exponential and algebraic forms. It is shown that there are some results about these properties of the strong solutions in L^∞-space.

Keywords: Two-component peakon system, Weakly dissipative, Asymptotic behavior.
2010 MSC: 35Q51, 35Q53, 35G25.

1. Introduction

In this paper, we would like to consider the following weakly dissipative modified two-component Dullin-Gottwald-Holm (mDGH2) system

$$\begin{cases} u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \beta u_{xxx} + \lambda(u - \alpha^2 u_{xx}) = \alpha^2(2u_x u_{xx} + uu_{xxx}) + \gamma\rho\bar{\rho}_x, & t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x + \lambda\rho = 0, \quad t > 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.1}$$

with weakly dissipative terms $\lambda(u - \alpha^2 u_{xx})$ and $\lambda\rho$, where $\rho = (1 - \partial_x^2)(\bar{\rho} - \bar{\rho}_0)$, and λ is a dissipative parameter.

For $\alpha = 1$, $\gamma = -1$ and $\beta = c_0 = \lambda = 0$, mDGH2 system (1.1) becomes the modified Camassa-Holm (mCH2) system

$$\begin{cases} u_t - u_{xxt} + 3uu_x - 2u_x u_{xx} - uu_{xxx} + \rho\bar{\rho}_x = 0, \quad t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x = 0, \quad t > 0, \quad x \in \mathbb{R}. \end{cases} \tag{1.2}$$

The mCH2 system (1.2) does admit peaked solutions in the velocity and average density, we refer this to Ref. [5] for details. In [5], the authors analytically identified the steepening mechanism that allows the singular solutions to emerge from smooth spatially confined initial data.

For $\alpha = 1, \beta = 0, \gamma = 0, c_0 = k, \lambda = 0$ and $\rho = 0$, the mDGH2 system (1.1) becomes the CH equation [1, 2]

$$u_t - u_{xxt} + ku_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0, \quad x \in \mathbb{R}, \tag{1.3}$$

which was derived with bi-Hamiltonian structure by Fokas and Fuchssteiner in [4]. In [3], Constantin and Lannes mathematically rigorously described the hydrodynamical relevance of CH equation (1.3).

For $\rho = 0$, the mDGH2 system (1.1) becomes the weakly dissipative DGH equation

$$u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \beta u_{xxx} + \lambda(u - \alpha^2 u_{xx}) = \alpha^2(2u_x u_{xx} + uu_{xxx}), \quad t > 0, \quad x \in \mathbb{R}. \tag{1.4}$$

In [7], Novruzov demonstrated the simple conditions on the initial data that lead to blow-up of the solution to the weakly dissipative DGH equation (1.4) in finite time.

Let $y = u - \alpha^2 u_{xx}$ and $v = \bar{\rho} - \bar{\rho}_0$. The mDGH2 system (1.1) admits the following form

$$\begin{cases} y_t + 2d_0 u_x + 2u_x y + uy_x - \gamma\rho v_x + ky_x + \lambda y = 0, \quad t > 0, \quad x \in \mathbb{R}, \\ \rho_t + (\rho u)_x + \lambda\rho = 0, \quad t > 0, \quad x \in \mathbb{R}, \end{cases} \tag{1.5}$$

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