Accepted Manuscript

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Nguyen Huy Tuan, Tran Dong Xuan, Nguyen Anh Triet, Daniel Lesnic

PII: S0893-9659(18)30081-8
DOI: https://doi.org/10.1016/j.aml.2018.03.016
Reference: AML 5464

To appear in: *Applied Mathematics Letters*

Received date : 7 December 2017
Revised date : 16 March 2018
Accepted date : 16 March 2018

Please cite this article as: N.H. Tuan, T.D. Xuan, N.A. Triet, D. Lesnic, On the Cauchy problem for a semilinear fractional elliptic equation, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.03.016

On the Cauchy problem for a semilinear fractional elliptic equation

Nguyen Huy Tuan[a], Tran Dong Xuan[b], Nguyen Anh Triet[c], Daniel Lesnic[d,*]

[a]*Applied Analysis Research Group, Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh City, Viet Nam*
[b]*Institute of Fundamental and Applied Sciences, Duy Tan University, 3 Quang Trung, Danang City, Vietnam*
[c]*Department of Mathematics, University of Architecture of HoChiMinh City, HoChiMinh City, Vietnam*
[d]*Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK*

Abstract

We study, for the first time in the literature on the subject, the Cauchy problem for a semilinear fractional elliptic equation. Under an *a priori* assumption on the solution, we propose the Fourier truncation method for stabilizing the ill-posed problem. A stability estimate of logarithmic type is established.
Keywords and phrases: Fourier regularization method; Cauchy problem; fractional elliptic equation; error estimate.
Mathematics subject Classification 2000: 35K05, 35K99, 47J06, 47H10

1. Introduction

Fractional differential equations arise in many fields of science and engineering [11], and most of the previous studies have been devoted to fractional diffusion and wave equations [3, 10, 13]. More recently fractional elliptic equations have become the point of interest of some distinguished studies [1, 2, 6] and the present paper is aimed to contribute towards broadening the overall understanding of inverse problems associated to equations of this type. In this paper, we consider the boundary value problem for the semilinear fractional elliptic equation

$$\partial_t^\alpha u + \Delta u = F(x,t,u(x,t)), \qquad (x,t) \in \Omega \times (0,T) =: Q_T, \tag{1.1}$$

with the following boundary conditions:

$$\begin{cases} u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = f(x), & x \in \Omega, \\ u_t(x,0) = g(x), & x \in \Omega, \end{cases} \tag{1.2}$$

where $\Omega \subset \mathbb{R}^d$ ($d = 1, 2, 3$) is a bounded domain with a smooth boundary $\partial\Omega$, and $T > 0$ is a given number. In (1.1), $\alpha \in (1,2)$ is the fractional order and ∂_t^α denotes the Caputo fractional derivative with respect to t, (see [9, 12]),

$$\partial_t^\alpha u(x,t) := \frac{1}{\Gamma(2-\alpha)} \int_0^t (t-s)^{1-\alpha} \frac{\partial^2 u}{\partial s^2}(x,s) ds, \quad (x,t) \in Q_T,$$

where Γ is the Gamma function. We note that a modified equation to (1.1) as

$$\partial_t^\alpha u - \Delta u = F(x,t,u(x,t)), \qquad (x,t) \in Q_T, \tag{1.3}$$

called a semilinear fractional wave equation, subject to the conditions (1.2) has been studied in [9].

In the case $\alpha \searrow 1$, the problem (1.1) becomes an ill-posed backward problem for the parabolic heat equation [14], whilst in the case of $\alpha \nearrow 2$, the problem (1.1) becomes a classical elliptic inverse problem (called the Cauchy problem for the Laplace equation), [7]. It is well-known that this latter problem is ill-posed in the sense of Hadarmard and regularization results have been obtained in [15]. A natural question is whether the Cauchy problem for the

*Corresponding author
Email addresses: nguyenhuytuan@tdt.edu.vn (Nguyen Huy Tuan), trandongxuan@gmail.com (Tran Dong Xuan), nguyenanhtriet@gmail.com (Nguyen Anh Triet), amt5ld@maths.leeds.ac.uk (Daniel Lesnic)