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Adnan Khaliq, Mujeeb ur Rehman

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On variational methods to non-instantaneous impulsive fractional differential equation

Adnan Khaliq*, Mujeeb ur Rehman

School of Natural Sciences,
National University of Sciences and Technology, Islamabad Pakistan

Abstract

In this paper by using the variational methods for a class of impulsive differential equation of fractional order with non-instantaneous impulses, we setup sufficient conditions for the existence and uniqueness of weak solutions. The problem is reduced to an equivalent form such that the weak solutions of the problem are defined as the critical points of a functional. Main results of the present work are established by using Lax-Milgram Theorem.

Keywords: Fractional differential equations, Variational methods, Lax-Milgram theorem, Non-instantaneous impulse.

1. Introduction

Fractional calculus generalizes the integer order integration and differentiation concepts to an arbitrary (real or complex) order. Fractional calculus is the most well known and valuable branch of mathematics which gives a good framework for biological and physical phenomena, mathematical modeling of engineering etc. To get a couple of developments about the theory of fractional differential equations, one can allude to the monographs of Lakshmikanthem et al. [4], Kilbas et al. [2], Pudlubny [6], Tarasov [7] and the references in there.

The impulsive differential equations have become very useful tools to portray the change of processes in which discontinuous jumps (impulsive conditions) abruptly appear. Such processes are normally seen in engineering, biology and physics. A maximum number of research papers are written on the existence theory of the solutions for the impulsive differential equations with instantaneous impulses [9, 16, 18]. But in many cases it has been noted that certain dynamics of evolution processes cannot delineate by the instantaneous impulses, for example: High or low level of glucose, Pharmacotherapy, this situation can be illustrated as an impulsive jump starting suddenly at any fixed point and remains continue on a finite interval of time. Such sort of systems are more useful to study the dynamics of evolution processes and are called non-instantaneous impulsive systems.

Hernadez & O'Regan introduced non-instantaneous impulsive differential equations in [14]. Recently, many authors applied nonlinear analysis methods (e.g. fixed point theory etc), to obtain the results dealing with the impulsive differential equations with non-instantaneous impulses for the existence and multiplicity of their solutions [5, 8, 11, 12, 20]. However, the problems (e.g. discussing BVP) for which the equivalent integral equation is not easy to obtain, these

*Corresponding author. Tel +92 322 6623874
Email addresses: adnankhaliqsns@gmail.com (Adnan Khaliq), mujeeburrehman345@yahoo.com (Mujeeb ur Rehman)