Accepted Manuscript

On a hyperbolic perturbation of a parabolic initial–boundary value problem

Boris N. Chetverushkin, Alexander A. Zlotnik

PII: S0893-9659(18)30092-2
DOI: https://doi.org/10.1016/j.aml.2018.03.027
Reference: AML 5475

To appear in: *Applied Mathematics Letters*

Received date : 23 January 2018
Revised date : 23 March 2018
Accepted date : 23 March 2018

On a hyperbolic perturbation of a parabolic initial-boundary value problem

Boris N. Chetverushkin[a], Alexander A. Zlotnik[b] [1]

[a] Keldysh Institute of Applied Mathematics, Miusskaya sq. 4a, 125047 Moscow, Russia

[b] National Research University Higher School of Economics, Myasnitskaya 20, 101000 Moscow, Russia

Abstract. We deal with a linear parabolic initial-boundary value problem and its hyperbolic perturbation with a small parameter $\varepsilon > 0$ in front of the 2nd order time derivative. We derive bounds for the perturbation error of the orders $O(\varepsilon)$ and $O(\sqrt{\varepsilon})$ in several norms in dependence with smoothness of data (without a priori conditions on solutions) but for non-smooth coefficients and keeping the free terms in equations as distributions. They essentially complement or improve some previously known bounds. In addition, we discuss regularizations of the initial time derivative.

Keywords: parabolic initial-boundary value problem, hyperbolic perturbation, perturbation error, energy norm

1. Introduction

We consider a linear parabolic initial-boundary value problem and its hyperbolic perturbation with a small parameter $\varepsilon > 0$ in front of the 2nd order time derivative. We derive error bounds between their weak solutions (i.e., for the perturbation error) of the orders $O(\varepsilon)$ and $O(\sqrt{\varepsilon})$ in several norms, including the parabolic and hyperbolic energy norms and a weaker norm, in dependence with smoothness of the initial data and free terms in equations of the problems (without a priori conditions on solutions) but for non-smooth coefficients and keeping the free terms as distributions. These error bounds essentially complement or improve previously known ones from [5] given in a rather general statement but only in the parabolic energy norm and without the distributional free terms and [8, 10] given under stronger a priori assumptions on the solution to the parabolic problem.

We do not aspire to the most general statement of our results for more clarity and brevity and confine ourselves by the equations without lower order terms and with time-independent coefficients as well as by the homogeneous Dirichlet boundary condition. But we apply only various versions of general energy techniques, and thus it is clear that the results could be generalized to much more general situations.

We also consider possible implicit elliptic and explicit by averaging regularizations for the initial time derivative to avoid the loss in regularity of the solution to the regularized problem.

Interest to perturbations of the discussed type has a long history that goes back to B. Riemann and finds applications in heat conductivity and thermoelasticity [9]. Recently it has resumed again and increased in connection with new approaches to numerical solution of problems in gas- and hydrodynamics and filtration on high-performance computers [2]-[4], [13].

2. The parabolic initial-boundary value problem and its hyperbolic perturbation

We deal with the linear parabolic initial-boundary value problem

$$\mathcal{P}u := \rho\partial_t u - \operatorname{div}(A\nabla u) = \operatorname{div}\mathbf{f} + f \quad \text{in} \quad Q := Q_T = \Omega \times (0,T), \quad u|_{\Gamma_T} = 0, \quad u|_{t=0} = u_0 \tag{1}$$

[1]Corresponding author.
E-mail addresses: office@keldysh.ru (B. Chetverushkin), azlotnik@hse.ru (A. Zlotnik)