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N. Bondarenko and V. Yurko

Abstract. We consider second-order functional differential operators with a constant delay. Properties of their spectral characteristics are obtained and a nonlinear inverse problem is studied, which consists in recovering the operators from their spectra. We establish the uniqueness and develop a constructive algorithm for solution of the inverse problem.

Keywords: differential operators, deviating argument, nonlinear inverse spectral problem.

AMS Mathematics Subject Classification (2010): 34A55 34K10 34K29 47E05 34B10 34L40

1. Introduction

In various real-world processes, the future behavior of the system depends not only on its present state and rate of change of the state (corresponding to the values of the function and its derivatives at the current point), but also on the states in the past. Such processes are described by functional differential equations with delay, which arise in physics, biology and especially in engineering and control theory (see the monographs [1-2] and the references therein).

This paper concerns an inverse spectral problem for the Sturm-Liouville equation with a constant delay. Inverse problems of spectral analysis consist in recovering operators from their spectral characteristics. Basic results of inverse problem theory for the classical differential Sturm-Liouville equation are contained in the monographs [3-6]. However, operators with a constant delay appear to be more difficult for investigation, and inverse problems for them are studied only in a few special cases (see [7-11]). The standard methods of inverse problem theory, applicable to differential operators (transformation operator method, method of spectral mappings, etc.), do not work for operators with delay, so one needs new approaches to construct a general spectral theory for the latter ones.

In this paper, we consider the boundary value problems $L_j(q)$, $j = 0, 1$, of the form

$$-y''(x) + q(x)y(x-a) = \lambda y(x), \quad x \in (0, \pi), \quad (1)$$

$$y(0) = y^{(j)}(\pi) = 0, \quad (2)$$

where λ is the spectral parameter, $a \in (0, \pi)$, $q(x)$ is a complex-valued function such that $q(x) \in L(a, \pi)$, and $q(x) \equiv 0$ for $x \in [0, a]$. We study the inverse spectral problem of recovering the potential $q(x)$ from the spectra of $L_j(q)$. More precisely, let $\{\lambda_{nj}\}_{n \geq 1}$, $j = 0, 1$, be the eigenvalues of $L_j(q)$. The inverse problem is formulated as follows.

Inverse problem 1. Given $\{\lambda_{nj}\}_{n \geq 1}$, $j = 0, 1$, construct $q(x)$.

Note that in the case of “large delay” when $a \geq \pi/2$, the characteristic functions of $L_j(q)$ depend linearly on the potential $q(x)$, i.e. the inverse problem becomes linear. This linear case was studied in [9] and [11]. For $a < \pi/2$ the characteristic functions depend nonlinearly on the potential, i.e. the inverse problem becomes nonlinear. This nonlinear case is essentially more difficult for investigating and constructing the global solution of the inverse problem. In this paper we consider the nonlinear case. For definiteness, let $a \in [2\pi/5, \pi/2)$. The case $a < 2\pi/5$ requires a separate investigation. In this paper we obtain a global constructive procedure for the solution of the inverse problem and establish its uniqueness. The main results of the paper are Theorem 1 and Algorithm 1 (see Section 3 below).

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