

## Accepted Manuscript

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PII: S0893-9659(18)30107-1  
DOI: <https://doi.org/10.1016/j.aml.2018.04.001>  
Reference: AML 5482

To appear in: *Applied Mathematics Letters*

Received date: 14 January 2018  
Revised date: 1 April 2018  
Accepted date: 1 April 2018

Please cite this article as: H. Bae, Analyticity of the inhomogeneous incompressible Navier–Stokes equations, *Appl. Math. Lett.* (2018), <https://doi.org/10.1016/j.aml.2018.04.001>

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# ANALYTICITY OF THE INHOMOGENEOUS INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

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ABSTRACT. In this paper, we obtain analyticity of the inhomogeneous Navier-Stokes equations. The main idea is to use the exponential operator  $e^{\phi(t)|D|}$ , where  $\phi(t) = \delta - \theta(t)$ ,  $\delta > 0$  is the analyticity radius of  $(\rho_0 - 1, u_0)$ , and  $|D|$  is the differential operator whose symbol is given by  $\|\xi\|_{l^1}$ . We will show that for sufficiently small initial data, solutions are analytic globally in time in critical Besov spaces.

## 1. INTRODUCTION

In this paper, we study the inhomogeneous incompressible Navier-Stokes equations governing the time evolution of the density  $\rho$ , the velocity  $u$  and the pressure  $\pi$  in  $\mathbb{R}^3$ :

$$\rho_t + u \cdot \nabla \rho = 0, \quad (1.1a)$$

$$\rho(u_t + u \cdot \nabla u) + \nabla \pi - \Delta u = 0, \quad \operatorname{div} u = 0. \quad (1.1b)$$

For the mathematical background of this model, see [9]. When  $\rho_0$  is away from zero, we take the density  $\rho$  which is a small perturbation of the constant density, say 1. In this case, we take  $\eta = 1 - \frac{1}{\rho}$  and rewrite (1.1) in terms of  $(u, \eta, \pi)$  as

$$\eta_t + u \cdot \nabla \eta = 0, \quad (1.2a)$$

$$u_t - \Delta u + \nabla \pi = -u \cdot \nabla u - \eta \nabla \pi + \eta \Delta u, \quad \operatorname{div} u = 0. \quad (1.2b)$$

We note that (1.2) satisfies the scaling invariant property: if  $(u, \eta, \pi)$  solves (1.2), so does

$$u_\lambda(t, x) = \lambda u(\lambda^2 t, \lambda x), \quad \eta_\lambda(t, x) = \eta(\lambda^2 t, \lambda x), \quad \pi_\lambda(t, x) = \lambda^2 \pi(\lambda^2 t, \lambda x).$$

In this paper, we choose the following scaling invariant Besov spaces for initial data

$$u_0 \in \dot{B}_{p,1}^{\frac{3}{p}-1}, \quad \eta_0 \in \dot{B}_{p,1}^{\frac{3}{p}}. \quad (1.3)$$

We require that  $\eta$  stay in a Banach algebra  $\dot{B}_{p,1}^{\frac{3}{p}}$  to deal with the product of  $\eta$  with  $u$  and  $p$ . Let

$$\begin{aligned} \mathcal{I}_0 &= \|u_0\|_{\dot{B}_{p,1}^{\frac{3}{p}-1}} + \|\eta_0\|_{\dot{B}_{p,1}^{\frac{3}{p}}}, \\ \|(u, \eta, \pi)\|_{\mathcal{E}_T} &= \|u\|_{\widetilde{L}_T^\infty \dot{B}_{p,1}^{\frac{3}{p}-1}} + \|u\|_{L_T^1 \dot{B}_{p,1}^{\frac{3}{p}+1}} + \|\eta\|_{L_T^\infty \dot{B}_{p,1}^{\frac{3}{p}}} + \|\pi\|_{L_T^1 \dot{B}_{p,1}^{\frac{3}{p}}}, \end{aligned} \quad (1.4)$$

where the regularity of  $\pi$  is determined by the following elliptic PDE:

$$-\Delta \pi = \operatorname{div}(u \cdot \nabla u + \eta \nabla \pi + \eta \Delta u). \quad (1.5)$$

We here restrict take  $p < 3$  for the well-posedness of (1.2): when  $p \geq 3$ ,  $\frac{3}{p} - 1$  is nonpositive and so it is impossible to bound  $\eta \nabla \pi$  and  $\eta \Delta u$  in the right-hand side of (1.2b) in  $\mathcal{E}_T$ .

*Date:* April 1, 2018.

*2010 Mathematics Subject Classification.* Primary 76D09; Secondary 76D03, 35Q35.

*Key words and phrases.* Inhomogeneous Navier-Stokes equations, Critical spaces, Gevrey estimates.

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