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Error Analysis of the Immersed Interface Method for Stokes Equations with an Interface

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Abstract

The immersed interface method using the three Poisson equation approach has been successfully developed to solve incompressible Stokes equations with interfaces [6, 8]. While the numerical results show second order convergence for both velocity and pressure, rigorous error analysis is still missing. Based on recent theoretical development, particularly the error analysis by Beale & Layton [1], second order convergence has been shown in this paper for both pressure and velocity under some assumptions.

Keywords:
Stokes equations, error analysis, finite difference method, immersed interface method, divergence-free
2010 MSC: 65N06, 65N15, 76D07

1. Introduction

Consider the incompressible Stokes equations in two dimensional spaces,

$$\nabla p = \mu \Delta \boldsymbol{u} + F + \boldsymbol{g}, \quad \nabla \cdot \boldsymbol{u} = 0, \quad \boldsymbol{x} \in \Omega, \quad \boldsymbol{u}|_{\partial\Omega} = \mathbf{0}, \tag{1}$$

where $\boldsymbol{u}$ is velocity, p is pressure, μ is coefficient of viscosity, $\boldsymbol{g}$ is a body force, F is a source distribution along a closed smooth interface $\Gamma \in \Omega$ as in Peskin's immersed boundary (IB) model [9], $F(\boldsymbol{x}) = \int_\Gamma f(s)\delta_2(\boldsymbol{x} - \boldsymbol{X}(s))ds$, where δ_2 is the 2D Dirac delta function and $\boldsymbol{x} = (x, y) \in \Omega^+ \cup \Omega^-$. The interface Γ is assumed to be closed in C^2 and has the parametric form $\boldsymbol{X}(s) = (x(s), y(s))$, where s is a parameter such as the arc-length and divides the domain into two disjoint sub-regions Ω^+ and Ω^-. The pressure, its normal derivative, and the normal derivative of the velocity are discontinuous across the interface. We will assume μ is constant, which can be written as $\mu = 1$ after scaling.