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Solving nonlinear singularly perturbed problems by fractional order exponential trial functions

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Abstract

The second-order time-varying nonlinear singularly perturbed problem is transformed to a weak-form integral equation by using the adjoint test functions. The weak-form integral equation method together with the fractional order exponential trial functions as the bases provides accurate numerical solutions of nonlinear singularly perturbed problems, of which the iterative algorithm is convergent very fast.

Keywords: Nonlinear singularly perturbed problem, Weak-form integral equation method, Adjoint test functions, Fractional order exponential trial functions

1. Introduction

Let us consider a time-varying second-order nonlinear singularly perturbed boundary value problem (SPBVP):

$$\varepsilon\ddot{y}(t)+b(t)\dot{y}(t)+c(t)y(t)=f(t,y,\dot{y}),\quad 0<t<t_f, \tag{1}$$

$$y(0)=\alpha,\quad y(t_f)=\beta, \tag{2}$$

where ε is a small parameter, and $f(t,y,\dot{y})$ is a time-dependent function and nonlinear in y and $\dot{y}$.

2. Adjoint test functions

For the use in later, the adjoint test function $v(t)$ is solved from

$$\mathcal{L}^*v(t):=\varepsilon\ddot{v}(t)-b(t)\dot{v}(t)-\dot{b}(t)v(t)+\lambda v(t)=0,\quad v(0)=v(t_f)=0. \tag{3}$$

Theorem 1. *From Eq. (3) the adjoint test functions $v_j(t)$ are found to be*

$$v_j(t)=\exp\left(\frac{\int_{t_f}^{t}b(\xi)d\xi}{2\varepsilon}\right)\sin\frac{j\pi t}{t_f}, \tag{4}$$

where

$$\lambda_j(t)=\frac{b^2(t)}{4\varepsilon}+\frac{\dot{b}(t)}{2}+\frac{\varepsilon j^2\pi^2}{t_f^2},\quad j\in\mathbb{N}. \tag{5}$$