# Accepted Manuscript


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PII: S0893-9659(18)30116-2
DOI: https://doi.org/10.1016/j.aml.2018.03.035
Reference: AML 5489

To appear in: *Applied Mathematics Letters*

Received date : 4 February 2018
Revised date : 23 March 2018
Accepted date : 25 March 2018

# Existence of solution of a forest fire spread model[☆]

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**Abstract**

We consider a wildfire spread model represented by the system (1). We use results from the theory of Hamilton-Jacobi equations to prove that there exists a classical solution of (1) for any $(\phi, t) \in \mathbb{R} \times (0, T)$ and some $T > 0$ and satisfies particular initial conditions. We also use the method of characteristics to obtain the solution of (1) in a certain form.

*Keywords:* wildfire spread model, Hamilton-Jacobi equations

*2010 MSC:* 35F21, 35F25, 70H20

## 1. Introduction

In this paper, we study the system

$$x_t = f_1(x_\phi, y_\phi), \quad y_t = f_2(x_\phi, y_\phi) \quad \text{for } (\phi, t) \in \mathbb{R} \times (0, T), \tag{1}$$

where $x_t$, $y_t$, $x_\phi$ and $y_\phi$ denote partial derivatives with respect to $t$ and $\phi$, respectively, of functions $x(\phi, t)$ and $y(\phi, t)$, and

$$f_1(x, y) = \frac{a^2 \cos\theta(x \sin\theta + y \cos\theta) - b^2 \sin\theta(x \cos\theta - y \sin\theta)}{\sqrt{a^2(x \sin\theta + y \cos\theta)^2 + b^2(x \cos\theta - y \sin\theta)^2}} + c \sin\theta,$$

$$f_2(x, y) = \frac{-a^2 \sin\theta(x \sin\theta + y \cos\theta) - b^2 \cos\theta(x \cos\theta - y \sin\theta)}{\sqrt{a^2(x \sin\theta + y \cos\theta)^2 + b^2(x \cos\theta - y \sin\theta)^2}} + c \cos\theta.$$

For simplicity, we just write $x$ and $y$ instead of $x(\phi, t)$ and $y(\phi, t)$ in (1). We note that $\phi$ is a parametrization variable of the curve $(x(\phi, t), y(\phi, t))$, so it does not appear in (1). Here $T > 0$ and $a, b, c, \theta$ are constants. This system was derived in [7] as a model of forest fire spread and its solution $(x, y)$ determines the position of wildfire in time. The ability to predict the direction and speed of fire is very important, since it is essential to make decisions regarding the fire extinguishing, evacuations of communities etc. Of course, such predictions are not easy, since the fire heavily depends on many

[☆] M.F. was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0378 and by the Slovak Grant Agency VEGA No. 2/0153/16. J.P. was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0378 and by the Slovak Grant Agency VEGA No. 1/0347/18.

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