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The structure function for system reliability as predictive (imprecise) probability

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ABSTRACT

In system reliability, the structure function models functioning of a system for given states of its components. As such, it is typically a straightforward binary function which plays an essential role in reliability assessment, yet it has received remarkably little attention in its own right. We explore the structure function in more depth, considering in particular whether its generalization as a, possibly imprecise, probability can provide useful further tools for reliability assessment in case of uncertainty. In particular, we consider the structure function as a predictive (imprecise) probability, which enables uncertainty and indeterminacy about the next task the system has to perform to be taken into account. The recently introduced concept of 'survival signature' provides a useful summary of the structure function to simplify reliability assessment for systems with many components of multiple types. We also consider how the (imprecise) probabilistic structure function can be linked to the survival signature. We briefly discuss some related research topics towards implementation for large practical systems and networks, and we outline further possible generalizations.

1. Introduction

In the mathematical theory of reliability, the main focus is on the functioning of a system, given the structure of the system and the functioning, or not, of its components. The mathematical concept which is central to this theory is the *structure function*. For a system with m components, let state vector $\underline{x} = (x_1, x_2, \ldots, x_m) \in \{0, 1\}^m$, with $x_i = 1$ if the ith component functions and $x_i = 0$ if not. The labelling of the components is arbitrary but must be fixed to define $\underline{x}$. The structure function ϕ: $\{0, 1\}^m \rightarrow \{0, 1\}$, defined for all possible $\underline{x}$, takes the value 1 if the system functions and 0 if the system does not function for state vector $\underline{x}$. In this paper, as in most of the literature, attention is restricted to coherent systems, for which $\phi(\underline{x})$ is not decreasing in any of the components of $\underline{x}$, so system functioning cannot be improved by worse performance of one or more of its components (generalization to allow incoherent systems is possible but would make concepts and notation later in the paper more complex). Coherence of a system is further usually assumed to imply that all the system's components are relevant, meaning that the functioning or not of each component makes a difference to the functioning of the system for at least one set of states for the other components.

The structure function is a powerful tool for reliability quantification, but in practice there may be uncertain or unknown aspects related to a system's functioning which can be taken into account by a generalization of the structure function to a probabilistic structure function. A main motivation for this generalization is that the system may have to deal with a variety of tasks of different types, which put different requirements on the system. We focus then on a specific future task to be performed, calling it the 'next task', and take uncertainty about the type of this task into account by using probabilities over the different types of tasks, and by generalizing this to imprecise probabilities which enables uncertainty and indeterminacy to be included in the modelling. This approach is very flexible; it can even be used to include the possibility of a fully unknown type of task, which might for example be suitable to reflect possible unknown threats to the system. A further motivation comes from the fact that there may simply be too many uncertainties affecting the system's functioning, which cannot be modelled in detail due to lack of meaningful information or limited time for detailed analysis. Throughout, it should be kept in mind that the proposed (imprecise) probabilistic structure function generalized the classical structure function, and as such provides a more flexible tool for reliability quantification. If one strongly feels that one can always model scenarios in full detail then one can argue that this generalization is not required, in which case perhaps the interest in the probabilistic structure function would be merely from the perspective

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of a mathematical exercise. However, we believe that there are plenty of real-world scenarios that will benefit from the flexibility provided by the (imprecise) probabilistic structure function when compared to the special case of the deterministic structure function, ensuring that the contribution of this paper goes far beyond merely a mathematical exercise.

Section 2 presents the structure function as a, possibly imprecise, predictive probability. This section also includes some motivating examples. Section 3 explains how the (imprecise) probabilistic structure function can be incorporated into (lower and upper) survival signatures for efficient system reliability quantification [1]. Uncertainty with regard to the type of the next task is considered in Section 4, and illustrated in an extensive example in Section 5. The paper concludes with a discussion of some related aspects in Section 6.

2. The structure function as (imprecise) probability

A simple way to reflect uncertainty about the system's functioning given the state vector $\underline{x}$ is by defining the structure function as a probability, so ϕ: $\{0, 1\}^m \to [0, 1]$. We define $\phi(\underline{x})$ as the probability that the system functions for a specific state vector $\underline{x}$ and for the next task the system is required to perform. It should be emphasized that explicit focus on the next task is not necessary when generalizing the structure function in this way, but it provides a natural tool for further uncertain aspects which we discuss later. We will simply refer to this generalized structure function as *probabilistic structure function*, and for simplicity we keep using the same notation $\phi(\underline{x})$ which is reasonable as the classical deterministic structure function is a special case of the probabilistic structure functioning, only using probabilities 0 and 1. Note that one could similarly define a probabilistic structure function for a system that has to perform multiple future tasks, this is left as a topic for future consideration.

We wish to emphasize that considering the structure function as a probability differs essentially from the classical use of the structure function with randomness on whether or not the individual components function. The corresponding probability that the system functions with random functioning of the components is usually called the 'reliability function' [2]. The important novelty in this paper is that the system functioning can be uncertain for given states of the components, which can occur due to a range of practical circumstances. Combining the probabilistic structure function with randomness for the state of the components is a straightforward further step, simplified by the use of the survival signature, as also presented in this paper.

Let S denote the event that the system functions as required for the next task it has to perform, then

$$\phi(\underline{x}) = P(S|\underline{x}) \tag{1}$$

This generalization already enables an important range of real-world scenarios to be modelled in a straightforward way. Scenarios where the flexibility of the probabilistic structure function might be useful are, of course, situations where even with known status of the components, it is not certain whether or not the system functions, that is performs its task as required. This may be due to varying circumstances or requirements which may not be modelled explicitly, or may not even be fully known. It could also just be that, in principle, the function of the system could be determined with certainty but that constraints on time or access to experts may prevent this. As an example, one could consider a collection of wind turbines as one system, with the task to contribute to overall generation of a level of energy required to provide a specific area with sufficient electricity for a specific period of time (we can consider this to be the 'next task'). One could consider each wind turbine as a component (with several other types of components in the system, that is irrelevant for now). Even if one knows the number of functioning components at a particular time, factors such as the weather, the availability of other electricity generating resources for the network, and the specific electricity demand, can lead to uncertainty about whether or not the system meets the actual requirements. To fit with the established deterministic definition of the structure function one could define system functioning in far more detail, but this may be hard to do in practice. As another example, one could think about a network of computers which together form a system for complex computations, where its actual success in dealing with required tasks might be achieved with some computers not functioning, but with some lack of knowledge about the exact number of computers required to complete tasks of different types. A further motivating example is given at the end of this section.

The generalization from deterministic to probabilistic structure function, although mathematically straightforward, requires substantial information in order to assess the probabilities of system functioning for all possible state vectors $\underline{x}$. While this modelling might explicitly take co-variates into account, thus possibly benefitting from a large variety of statistical models, it may be difficult to actually formulate the important co-variates and one might not know their specific values. This leads to two further topics we wish to discuss, namely what precisely is meant when we say that the system functions, and a generalization of probability to allow lack of knowledge to be reflected.

Whether or not a real-world system performs its task may depend on many circumstances beyond the states of the system components. It may be too daunting to specify system functioning for all possible circumstances, and it may even be impossible to know all possible circumstances. Hence, speaking of 'system functioning' in the traditional theoretic way seems rather restricted. One suggestion would be to only define system functioning for one (or a specified number of) application(s), e.g. whether or not a system functions at its next required use. This will not be sufficient for all real-world scenarios, but it will enable important aspects of uncertainty on factors such as different tasks and circumstances to be taken into account. We believe that this is a topic that requires further attention, it links to many system dependability concepts including flexibility and resilience.

The generalization to a probabilistic structure function provides substantial enhanced modelling opportunities for system reliability and dependability. However, the use of single-valued probabilities for events does not enable the strength or lack of information to be taken into account, with most obvious limitation the inability to reflect if 'no information at all' is available about an event of interest. In recent decades, theory of *imprecise probability* [3] has gained increasing attention from the research community, including contributions to reliability and risk [4]. It generalizes classical, precise, probability theory by assigning to each event two values, a lower probability and an upper probability, denoted by $\underline{P}$ and $\overline{P}$, respectively, with $0 \le \underline{P} \le \overline{P} \le 1$. These can be interpreted in several ways; for the current discussion it suffices to regard them as the sharpest bounds for a probability based on the information available, where the lower probability typically reflects the information available in support of the event of interest and the corresponding upper probability reflects the information available against this event. The case of no information at all can be reflected by $[\underline{P}, \overline{P}] = [0, 1]$ while equality $\underline{P} = \overline{P}$ reflects perfect knowledge about the probability and results in classical precise probability as a special case of imprecise probability.

We propose the further generalization of the structure function within imprecise probability theory by introducing the *lower probabilistic structure function*