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Abstract

In this paper, we apply the augmented Lagrangian (AL) approach to steady buoyancy driven flow problems. Two AL preconditioners are developed based on the variable's order, specifically whether the leading variable is the velocity or the temperature variable. Correspondingly, two non-augmented Lagrangian (NAL) preconditioners are also considered for comparison. An eigenvalue analysis for these two pairs of preconditioners is conducted to predict the rate of convergence for the GMRES solver. The numerical results show that the AL preconditioner pair is insensitive with respect to the mesh size, Rayleigh number, and Prandtl number in terms of GMRES iterations, resulting in a significantly more robust preconditioner pair compared to the NAL pair. Accordingly, the AL pair performs much better than the NAL pair in terms of computational time. For the AL pair, the preconditioner with velocity as the leading variable gives slightly better efficiency than the one with temperature as the leading variable.

Keywords: augmented Lagrangian preconditioners, buoyancy driven flow problems, finite element method, eigenvalue analysis
2010 MSC: 65N30, 65N22, 76D05, 76E06

1. Introduction

Using the Boussinesq approximation the non-dimensional steady buoyancy driven flow equations

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} - \sqrt{\frac{Pr}{Ra}} \Delta \boldsymbol{u} + \nabla p + \hat{g} T = 0, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

$$\boldsymbol{u} \cdot \nabla T - \frac{1}{\sqrt{PrRa}} \Delta T = 0, \tag{3}$$

are derived on some open domain $\Omega \subset \mathbb{R}^2$ [1]. The symbols $\boldsymbol{u}$, p and T denote the velocity, pressure, and temperature variables, respectively. The Prandtl number Pr describes the ratio of momentum diffusivity to heat diffusivity, and the Rayleigh number Ra is the ratio of natural convection to conductive heat transfer. The quantity $\hat{g}$ is the unit vector along the gravity direction.

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