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Multiple positive solutions for a system of impulsive integral boundary value problems with sign-changing nonlinearities

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Abstract In this paper we investigate a system of impulsive integral boundary value problems with sign-changing nonlinearities. Using the fixed point theorem in double cones, we prove the existence of multiple positive solutions.
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Keywords: Sign-changing nonlinearity; Positive solution; System of impulsive equation; Fixed point theorem in double cones; Integral boundary condition

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1. Introduction

In this paper we consider the system of impulsive integral boundary value problems (BVPs)

$$\begin{cases} u_i''(t) + a_i(t)u_i'(t) + b_i(t)u_i(t) + c_i(t)f_i(t, u_1(t), u_2(t)) = 0, \quad t \in J', \\ -\Delta u_i'|_{t=t_k} = I_i^k(u_1(t_k), u_2(t_k)), \quad k = 1, 2, \ldots, m, \\ u_i(0) = \displaystyle\int_0^1 g_i(s)u_i(s)ds, \quad u_i(1) = \int_0^1 h_i(s)u_i(s)ds, \quad i = 1, 2, \end{cases} \tag{1.1}$$

where $J' = J \backslash \{t_1, t_2, \ldots, t_m\}$, $J = [0, 1]$, $0 = t_0 < t_1 < \ldots < t_m < t_{m+1} = 1$. For $i = 1, 2$, $a_i \in C(J)$ such that $\int_0^1 a_i(t)dt > 0$, $b_i \in C(J, (-\infty, 0))$, $c_i \in C(J, \mathbb{R}^+)$, $c_i(t) \not\equiv 0$, g_i, $h_i \in L^1(J, \mathbb{R}^+)$, $f_i \in C(J \times \mathbb{R}^+ \times \mathbb{R}^+, \mathbb{R})$ and $f_1(t, 0, u_2) \geq 0$, $f_2(t, u_1, 0) \geq 0$ for all $t \in J'$, $I_i^k \in C(\mathbb{R}^+ \times \mathbb{R}^+, \mathbb{R})$, where $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+ = [0, +\infty)$. $\Delta u_i'(t_k) = u_i'(t_k^+) - u_i'(t_k^-)$, where $u_i'(t_k^+)$ and $u_i'(t_k^-)$ are the right and left limits of $u_i'(t)$ at t_k.

The model of impulsive differential equation describes evolution processes in which their states change abruptly at certain moments in time. For an introduction to the basic theory of impulsive differential equations, we refer to [1-3] and the references therein. Considerable effort has been devoted to impulsive differential equations due to their theoretical challenge and potential applications, for example [4-11].

Hao, Liu and Wu [4] considered the following second order impulsive integral BVP:

$$\begin{cases} u''(t) + a(t)u'(t) + b(t)u(t) + \lambda c(t)f(t, u(t)) = 0, \quad t \in (0, 1) \setminus \{t_1, t_2, \cdots, t_m\}, \\ -\Delta u'|_{t=t_k} = \lambda I_k(u(t_k)), \quad k = 1, 2, \ldots, m, \\ u(0) = \displaystyle\int_0^1 g(s)u(s)ds, \quad u(1) = \int_0^1 h(s)u(s)ds, \end{cases}$$

where $f \in C(J \times \mathbb{R}^+, \mathbb{R}^+)$, $I_k \in C(\mathbb{R}^+, \mathbb{R}^+)$. Under different combinations of superlinearity and sublinearity of the nonlinear term and the impulses, various existence, multiplicity, and nonexistence results for positive solutions are derived in terms of the parameter λ lying in some interval. The argument is based on Krasnoselskii's fixed point theorem. In [10], the authors studied the following impulsive BVP:

$$\begin{cases} u''(t) - p(t)u'(t) - u(t) + \lambda f(t, u) + \mu g(t, u) = 0, \quad t \in [0, T] \setminus \{t_1, t_2, \cdots, t_m\}, \\ \Delta u'|_{t=t_k} = I_k(u(t_k)), \quad k = 1, 2, \ldots, m, \\ u(0) = u(T) = 0. \end{cases}$$

Existence results for multiple solutions are obtained by using variational methods combined with a three critical points theorem. Using the Leggett-Williams fixed point theorem, Henderson and

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