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Reductions of Darboux transformations for the $\mathcal{PT}$-symmetric nonlocal Davey–Stewartson equations

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ABSTRACT

In this letter, a study of the reductions of the Darboux transformations (DTs) for the $\mathcal{PT}$-symmetric nonlocal Davey–Stewartson (DS) equations is presented. Firstly, a binary DT is constructed in integral form for the $\mathcal{PT}$-symmetric nonlocal DS-I equation. Secondly, an elementary DT is constructed in differential form for the $\mathcal{PT}$-symmetric nonlocal DS-II equation. Afterwards, a new binary DT in integral form is also found for the nonlocal DS-II equation. Moreover, it is shown that the symmetry properties in the corresponding Lax-pairs of the equations are well preserved through these DTs. Thirdly, based on above DTs, the fundamental rogue waves and rational travelling waves are obtained.

1. Introduction

In the last several years, $\mathcal{PT}$-symmetric systems which allow for lossless-like propagation due to their balance of gain and loss have attracted considerable attention and triggered renewed interest in integrable systems. These nonlocal integrable equations are different from local integrable equations and could produce novel patterns of solution dynamics and intrigue new physical applications [1–17]. As an integrable multidimensional versions of the nonlocal nonlinear Schrödinger equation, a new integrable nonlocal Davey–Stewartson (DS) equation is recently introduced in Refs. [7,10]:

$$iu_t + \frac{1}{2}\alpha^2 u_{xx} + \frac{1}{2}u_{yy} + (uv - w)u = 0, \tag{1}$$

$$w_{xx} - \alpha^2 w_{yy} - 2\left[(uv)\right]_{xx} = 0, \tag{2}$$

where $v(x, y, t) = \epsilon\bar{u}(-x, -y, t)$, $\epsilon = \pm 1$. u, v and w are functions of x, y, t, $\alpha^2 = \pm 1$ is the equation-type parameter ($\alpha^2 = 1$ being the DS-I and $\alpha^2 = -1$ being DS-II). Here the sign $\bar{u}$ represents the complex

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conjugation of this function, and ϵ is the sign of nonlinearity. For this equation, the corresponding auxiliary linear system has the form as

$$L\Phi = 0, \quad L = \partial_y - J\partial_x - P, \tag{3}$$

$$M\Phi = 0, \quad M = \partial_t - \sum_{j=0}^{2} V_{2-j}\partial^j = \partial_t - i\alpha^{-1}J\partial_x^2 - i\alpha^{-1}P\partial_x - \alpha^{-1}V, \tag{4}$$

$$J = \alpha^{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; P = \begin{pmatrix} 0 & u \\ -v & 0 \end{pmatrix}, \; V = \frac{\mathrm{i}}{2}\begin{pmatrix} \omega_1 & u_x + \alpha u_y \\ -v_x + \alpha v_y & \omega_2 \end{pmatrix}, \tag{5}$$

with

$$w = uv - \frac{1}{2\alpha}(\omega_1 - \omega_2). \tag{6}$$

The compatibility condition means that $[L, M] = 0$ if and only if u, v, w satisfy the DS system.

For the local DS equations, several forms of Darboux transformations are given in [18–21]. Especially, for the local DS-II equation, a new reduction of DT is given in [21], and the solutions are expressed in terms of Grammian type determinants. In this $\mathcal{PT}$-symmetric nonlocal DS system, a binary DT is constructed for the nonlocal DS-I equation, and an elementary DT is obtained for the nonlocal DS-II equation. Moreover, inherited from the idea proposed in [21], a new binary DT in integral form for the nonlocal DS-II equation is also proposed under certain reductions.

This letter is organized as follows. In Section 2, for the $\mathcal{PT}$-symmetric nonlocal DS-II equation, an elementary DT in differential form and a new binary DT in integral form are constructed. In Section 3, for the $\mathcal{PT}$-symmetric nonlocal DS-I equation, the elementary DT is not enough, so we construct a binary DT in integral form. In addition, all the symmetry properties in the corresponding Lax-pairs are shown to be well preserved through these DTs. Moreover, as applications of these DTs, with certain reductions on the eigenfunctions and parameters in our DT procedure, some interesting solutions are obtained and discussed in Section 4, including rogue waves and rational travelling waves.

2. Darboux transformations for the $\mathcal{PT}$-symmetric nonlocal DS-II equation

As we know, the local DS-II equation possesses a Darboux transformation in differential form. Moreover, for the partially $\mathcal{PT}$-symmetric nonlocal DS-II equation, a differential form DT has been constructed in [12]. For this $\mathcal{PT}$-symmetric nonlocal DS-II equation, one can also construct Darboux transformation in the differential form.

It is already shown in [22] that for any invertible matrix θ such that $L(\theta) = M(\theta) = 0$, the operator

$$G_\theta = \theta\partial\theta^{-1}, \quad \partial = \partial_x, \tag{7}$$

makes L and M form invariant under the elementary Darboux transformation:

$$L \to \tilde{L} = G_\theta L G_\theta^{-1}, \; M \to \tilde{M} = G_\theta M G_\theta^{-1}.$$

The potential matrix P in (3)–(4) satisfies the following symmetric reduction

$$-\sigma_\epsilon P(x, y, t)\sigma_\epsilon^{-1} = \overline{P}(-x, -y, t), \quad \sigma_\epsilon = \begin{pmatrix} 0 & \epsilon \\ 1 & 0 \end{pmatrix}. \tag{8}$$

Considering the following zero curvature condition:

$$P_t - V_{2,y} + [P, V_2] + \Delta = 0, \tag{9}$$