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Solving a Class of Quadratic Matrix Equations

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Abstract

Let A be a matrix with $A^{-1} = A$. We find all commuting solutions and some non-commuting solutions of the matrix equaiton $AXA = XAX$. All non-commuting solutions are also obtained under a full rank condition.

Keywords: Matrix equation; idempotent matrix; spectral perturbation
AMS Subject Classifications: 15A24; 15A18

1 Introduction

In this paper, we try to solve the homogeneous quadratic matrix equation

$$AXA = XAX \tag{1}$$

with a given $n \times n$ complex matrix A. This equation is referred to as the *Yang-Baxter-like matrix equation* because of its analog in format to the classical Yang-Baxter equation [8] for the delta function Fermi gas and for an 8-vertex model.

There have been solutions [1, 2, 7] of (1) with some physical applications for various classes of A, most of which are commuting ones, that is, the solutions X satisfying $AX = XA$. In particular, all commuting solutions have been constructed in [4] when A is diagonalizable. However, finding all non-commuting solutions is very challenging.

At the end of our recent paper [5], we asked the question of finding all solutions of (1) with A satisfying $A^{-1} = A$. The purpose of this paper is to answer the above question.

In the next section, we first find all commuting solutions of (1) and characterize other ones by using the theory of Sylvester's equation. In Section 3, we seek solutions that can be represented by diagonal matrices. In Section 4 we use a spectral perturbation technique to find all non-commuting solutions under a full rank condition. We conclude in Section 5.

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