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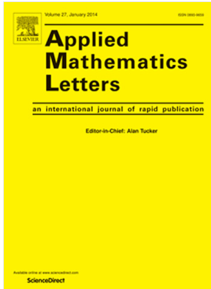

Optimizing and improving of the C-to-R method for solving complex symmetric linear systems

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Optimizing and improving of the C-to-R method for solving complex symmetric linear systems☆

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Abstract

For complex symmetric linear systems, Axelsson et al. (Numer. Algor., 66:811–841, 2014) proposed the C-to-R method. In this paper, by further studying the C-to-R method with W and T being symmetric positive semidefinite, the optimal iteration parameter for the C-to-R method $\alpha_{opt} = \frac{\sqrt{2\sqrt{2}}}{2}$ is obtained and the C-to-R method is optimized. Furthermore, based on the optimized C-to-R method, we further propose an optimized preconditioner. Eigenvalue properties of the optimized preconditioned matrix are analyzed, which show that all the eigenvalues of the preconditioned matrix are located in tighter interval. Numerical results are presented, not only confirm the validity of the theoretical analysis, but also demonstrate the feasibility and effectiveness of the proposed optimized C-to-R method.

Key words: preconditioner, optimal parameter, C-to-R method, complex symmetric linear system, spectral radius.
2000 MSC: 65F10, 65N20, 65F50, 65N22

1. Introduction

We focus our interests on the solution of the following complex symmetric linear system

$$\mathbf{A}u \equiv (W + \mathrm{i}T)z = c, \ \ \mathbf{A} = (W + \mathrm{i}T) \in \mathbb{C}^{n\times n} \ \text{ and } \ z, c \in \mathbb{C}^n. \tag{1.1}$$

where W and $T \in \mathbb{R}^{n\times n}$ are symmetric positive semidefinite (SPSD) matrices and satisfy $\mathrm{null}(W) \cap \mathrm{null}(T) = \{0\}$. $z := x + \mathrm{i}y$, $c := f + \mathrm{i}g$, $x, y, f, g \in \mathbb{R}^n$ and i denotes the imaginary unit. This system can be rewritten as a real form [2, 12]:

$$\mathscr{A}\tilde{z} \equiv \begin{pmatrix} W & -T \\ T & W \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix} \equiv b. \tag{1.2}$$

Such linear system (1.1) or (1.2) arise in many important applied problems, such as wave propagation [18], distributed control problems [16], structural dynamics [13], quantum mechanics [14], FFT-based solution of certain time-dependent PDEs [11, 21], molecular scattering and so forth. Fore more details, readers can consult [6, 19] and the references therein. Hence, there is a tremendous need for the fast solution to the complex linear system (1.1).

Recently, many efficient iteration methods and preconditioning techniques have been developed for solving the linear system (1.1) or (1.2). For instance, when W and T are SPSD and at least one of them is symmetric

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